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The development and evaluation of a basic skills mathematics curriculum for adult learners

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THE DEVELOPMENT AND EVALUATION OF A BASIC SKILLS MATHEMATICS CURRICULUM FOR ADULT LEARNERS

by
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ABSTRACT

Shirley Hofer and Annette M. Schenkel, The Development and Evaluation of a Basic Skills Mathematics Curriculum for Adult Learners, 1996, J. Sooy, Mathematics Education

The purpose of this study is to develop a basic skills mathematics curriculum for adult learners, evaluate student progress, and survey the instructors' opinions.

Gagne's curriculum model was used to develop a new curriculum addressing the problems of the traditional curriculum. Research was cited to substantiate each curriculum change. The new curriculum successfully addressed each of the concepts gathered from the related literature.

Student progress was evaluated at Gloucester County College from January to March of 1996. All of the nine MAT-010 classes used the new curriculum. A dependent t-test was applied to pretest and posttest scores of the New Jersey College Basic Skills Placement Tests for fifty-eight students. The difference in scores was significant at the .01 level. Results indicated that the students' computational achievement was significantly improved after covering the first two units of the text, which did not include computational instruction. These results concur with the NCTM Standards that concluded that remediation is more effectively taught by methods which stress understanding and not computational drill.

All of the five MAT-010 instructors were surveyed. The results of the opinionnaire showed that the instructors' opinions towards the new curriculum were favorable. This was determined by the use of the Likert Method.

MINI-ABSTRACT

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CHAPTER 1

Introduction to the Study

Introduction

This chapter presents an introduction to the development and evaluation of an innovative basic mathematics curriculum for adults. The motivating factors underlying this study are presented first, followed by the statement of the problem. The significance of the problem section explains the importance of addressing the specific needs of the adult remedial mathematics student. The limitations, assumptions, and procedures sections explain the researchers' methods of pursuing this endeavor. Definitions of terms unique to this study are also included.

Background

After teaching basic skills mathematics to adults over a period of several years, the researchers concurred that the curriculum that was in place at Gloucester County College was not meeting the unique needs of adult learners. The opportunity to develop a more appropriate curriculum came to the forefront when the researchers were enrolled in a curriculum development course and were required to submit ideas for a new curriculum of their choice. The researchers were convinced that it was vital to present remedial material in a new sequence in order to address the problems in the existing curriculum. In the process of formulating their ideas, they contacted various publishers of basic mathematics texts and perused a multitude of textbook brochures. Their search for a different sequence of topics was futile. Therefore, the researchers decided to incorporate their ideas for a new sequence of topics into a new curriculum that applied concepts gathered from research literature on adult education.

Statement of the Problem

The purpose of this study is to develop a basic skills mathematics curriculum for adult learners, evaluate student progress and survey the instructors' opinions.

Significance of the Problem

All community college students must pass a basic skills mathematics test to continue towards their degree. Passing this test is a requirement before they can get credit for, or even enroll in any non-developmental mathematics course. The curricula currently in place are not as effective as they could be towards this end. Jack Friedlander (1979) of California University stated in the Junior College Resource Review that although there has been much experimentation in remedial mathematics course formats and instructional formats, some existing problems persist. These problems are avoidance of remedial courses by students, high attrition rates, and low achievement levels. This is also evidenced by an abundance of research and literature indicating particular problems of adult learners. Current curricula do not take into account the special needs of the adult learners. They do not draw on the students' varied life experiences. The sequence of the topics and the methods of teaching are exactly the same as those used in the elementary levels. Instead, review material should be presented differently. Knowles (1980) shows in The Modern Practice of Adult Education: From Pedagogy to Andragogy that andragogical processes should be used in review situations as opposed to the pedagogical methods employed when materials are initially introduced. Research also suggests methods such as spiraling, teaching for understanding, "distributed practice", "connectedness", and "self-talk" to address problems of adult basic mathematics students. Appropriate curricular changes, however, have not been forthcoming. Because few, if any, curricular changes have been made, there is an absence of experimental research on these issues.

The intent and purpose of this specific basic skills mathematics curriculum is to address these problems by providing adult learners with the experiences necessary for their proficiency in basic mathematics using the methods suggested by research. A limited evaluation of this curriculum is also included.

Limitations of the Study

The evaluation of a new basic skills math curriculum was conducted at Gloucester County College, a southern New Jersey two year community college with an extensive remediation program, over a seven week period from January to March of 1996. Student enrollment is approximately 4400 with approximately a 9% minority population. All of the nine MAT-010 classes used the new curriculum for the spring 1996 semester. Five of the nine MAT-010 classes were evaluated after using the curriculum for approximately seven weeks. All of the five MAT-010 instructors were surveyed at the end of the seven week period.

The traditional basic skills mathematics curriculum covers computations with whole numbers, fractions, decimals and percents. The new curriculum covers the same material using a different sequence and approach.

Assumption

For the most part, for the purposes of this study it is assumed that the textbook, located in the appendix, is the newly developed curriculum.

Definition of Terms

andragogy - from the Greek word *aner* (meaning adult) - thus being defined as the art and science of helping adults (or, even better, maturing human beings) learn (Knowles, 1980)

connectedness - having value and meaning beyond the instructional context---a connection to the larger social context within which students live; exhibited in instruction when students address real-world public problems or use personal experiences as a context for applying knowledge (Newmann, 1993/1994)

distributed practice - practice in the form of either multiple presentations of the information to be learned (e.g., reviews) or in the form of tests; the practice sessions are distributed over a relatively lengthy periods of time (e.g., three reviews in three months) (Dempster, 1993/1994)

pedagogy - from the Greek words *paid* (meaning child) and *agogus* (meaning guide or leader)- thus being defined as the art and science of teaching children (Knowles, 1980)

self-talk - is anything one says to oneself. It can be positive, negative, encouraging, discouraging, uplifting, self-defeating, productive, or counterproductive. Positive self-talk is motivating and answer-seeking. Successful students use self-talk when they ask themselves questions about how to begin a problem, what result is desired, what information is given, etc.

spiral curriculum - a concept credited to Jerome S. Bruner that involves revisiting the same curricular content and expanding the level of mastery by building on previously learned ideas

Procedures

Gagne's curriculum model was used to develop a new curriculum (see Appendix A) addressing the problems of the traditional curriculum. Each curriculum change was justified. Research was cited to substantiate each change.

The curriculum was evaluated using an experiment. Five classes were given a version of the New Jersey Basic Skills Test as a pretest. Two of the four units of the new curriculum were covered in all of the classes. The first unit covered relationships of rational numbers while the second covered approximation skills, problem solving skills, and test taking strategies. Upon completion of the second unit, the five classes were given a second version of the New Jersey Basic Skills Test. A dependent t test was used to determine whether the first two units, which do not include any computational skills review, had affected student achievement.

In addition to the student experiment, all MAT-010 instructors were surveyed. This survey evaluated the sequencing of the first two units and the attitudes of the students by means of an opinionnaire.

CHAPTER 2

Review of Related Literature

Introduction

The researchers found that there is a definite absence of related research on remedial basic skills mathematics curricula for the adult learner. This absence of research may be attributed to the fact that most institutions rely solely on the textbook to define the developmental mathematics curriculum. Hence, the presentations in this chapter are only those of related literature. The material presented is divided into the following areas of discussion: (1) connectedness and teaching for understanding, (2) sequencing content, (3) distributed practice, (4) use of alternative methods and estimation to teach problem solving, (5) metacognition, self-talk, and overcoming mathematics anxiety, (6) clarity and language, and (7) humor, games, and cooperative learning.

Review of Related Literature

Connectedness and teaching for understanding. "Making connections is a fundamental component of a coherent curriculum. Finding connections among the students, subject-area content, and the outside world makes for more meaningful, coherent learning" (Pate, McGinnis, and Homestead, 1994, p. 62). Jerome S. Bruner (1977) equates learning how things are related to learning the structure of knowledge. Newmann and Wehlage (1993/1994) stress in their article, "Five Standards of Authentic Instruction", that when students address real-world problems or use their personal experiences as a context for problem solving, then instruction becomes connected. "Remediation is most effective when it occurs in relationship to a student's interest and when it supports his social, personal and vocational goals" (Sabatino & Mann, 1982, p. 49).

In his book, The Modern Practice of Adult Education, Malcolm Knowles (1980) suggests that remedial learning should be approached andragogically, making use of the learners' prior learning. The guidelines for adult education in Girl Scouting stipulate that each adult learner is unique and brings prior experience to the learning situation which should be respected and utilized (Preston, 1995).

According to Jere Brophy (1992/1994), current research focuses on the role of the student, recognizing that students try to make sense of information and relate it to what they already know. Students need to develop and link new knowledge to preexisting knowledge and beliefs and to anchor the newly acquired knowledge in concrete experiences. These methods will enable the students to get beyond the rote memorization of rules to achieve understanding. Bruner (1977) suggests that the ability of students to generalize develops from the understanding of a subject, and that students should strive continually to relate newly acquired information to the subject. According to Sheila Tobias (1993), mathematics is usually taught in fragmented bits by teachers who were taught the same way. Students are tested on the discrete bits, never realizing how these pieces of information are integrated.

In agreement with Brophy's theories, Tobias (1993) writes that students long to understand facts in context, to find connections, and to comprehend underlying structures. The rules are minimized as students are shown the connectedness of the content. Using historical facts or relating an example to a student's life experiences, thereby making the concept part of long-term memory, is a more effective alternative to memorizing rules. "The facts must be presented in some connection and in some sort of system, since isolated items are laboriously acquired and easily forgotten" (Polya, 1990, p. 218).

The following quotation from Polya (1990) emphasizes the importance of understanding versus memorization. "To apply a rule to the letter, rigidly, unquestioningly, in cases where it fits and in cases where it does not fit, is pedantry. . . . Some pedants are quite successful; they understood their rule, at least in the beginning (before they became pedants). . . . And if you are inclined to be a pedant and must rely upon some rule learn this one: Always use your own brains first" (p. 148-149).

Sequencing content. Bruner (1977) advocates use of a spiral curriculum to effectively teach basic mathematical ideas. It should be structured like a funnel. "To be in command of these basic ideas, to use them effectively, requires a continual deepening of one's understanding of them that comes from learning to use them in progressively more complex forms" (p. 13). John Dewey (1938) had even used this same metaphor to describe how to organize subject matter. He suggested that facts and ideas "become the ground for further experiences in which new problems are presented. The process is a continual spiral" (p. 79). Bruner suggests that the curriculum cultivates the student's mathematical intuition and allows the student to revisit the same curricular content, thereby expanding the student's level of mastery. Oliver (1965) suggests that each revisit be considered as a loop in the spiral, differing from the former loop both in depth and perspective.

The sequencing of the curriculum should depend primarily on the knowledge and experience of the students (Houle, 1972). Pedagogical models of learning are not appropriate for adult remedial or review situations. The logical sequence of the curriculum, which is necessary in the pedagogical model, should be replaced by a sequence prompted by readiness (Knowles, 1980). "Adults, whose conceptual equipment is already fairly sophisticated, might best learn elementary mathematics the second time around by diving in somewhere, anywhere at all, and, assisted by an informed interlocutor, proceed in ever-widening concentric circles" (Tobias, 1993, p. 168). Tobias likens the mathematical links missing from most adult remedial students' understanding to dropped stitches in a knitted garment. She believes that adults should be able to pick up the lost stitches without having to knit the entire garment again. Teachers of adults must assume that they are "experienced, able to think for themselves, and eager to understand" (p.168).

Distributed practice. Dempster (1993/1994) is a proponent of distributed or spaced practice, either multiple presentations of the material or multiple presentations of tests.

> "Research has shown that, under certain conditions, practice may either reduce the effects of interference or result in proactive or retroactive facilitation of learning. For example, the acquisition of skill in multiplication is normally hampered by brief exposures to problems with similar or identical digits and products, because problems encountered early in a sequence interfere with problems introduced later and vice versa. But with continued practice on both old problems and new problems, these difficulties can be avoided" (p. 204).

The NCTM Standards (1989) suggest "the systematic maintenance of student learnings," while opposing "extended periods of individual seatwork practicing routine tasks" and "rote memorization of facts and procedures" (p. 129). The efficacy of distributed practice is evidenced by research at all instructional levels. Walberg (1988) reported that spaced practice interspersed with other activities is superior to equal amounts of time devoted to massed practice.

> "Although 'massed' practice, which occurs over a relatively brief period (of time), may result in rapid acquisition of new material, the learning is not as durable or as resistant to interference as that acquired through frequent distributed practice. Research suggests that distributed practice does more than simply increase the amount learned; it frequently shifts the learner's attention away from the verbatim details of the material being studied to its deeper conceptual structure" (Dempster, 1993/1994, p. 204).

Use of alternate methods and estimation to teach problem solving. In her writings, Sheila Tobias (1993) points out that life experiences can be used to develop methods of doing mathematical problems. She proposes that students need to be encouraged to use personal reference points and intuition to restructure problems so that they make sense. From the very earliest of grades, intuition is discouraged and using a student's knowledge of his own world is almost never tapped as a resource. "Intuition can be developed like any other skill. It responds to exposure to math and to other related experiences" (p.143).

In Overcoming Math Anxiety, Tobias (1993) makes the point that teachers do their students a very big disservice by portraying themselves as infallible, always able to come up with the correct answer easily and without any error along the way, even appearing sometimes to pull the answer right out of the air. If a student does not understand how the teacher came up with the correct result, it sometimes leads the student to believe he is incapable of ever solving these types of problems, reinforcing his already suffering self-esteem.

Instruction at the elementary school level has fostered the notion of one and only one method to solve each problem. Different methods should be actively encouraged (Tobias, 1993). Brophy (1992/1994) suggests that students should be encouraged to develop their own explanations, make predictions, and debate alternative approaches to problems. Tobias (1993) also notices that most adults are ashamed of any methods they devise on their own to solve problems, assuming them to be inferior to the "right method", thus rendering them useless.

Remedial students do not need more rules to memorize, they need fewer rules and more understanding. A better approach would be to relate the concept to something the student already has in his long-term memory. Understanding a problem would eventually lead the student to come up with a useful algorithm or rule of his own, possibly differing from the generally accepted rule, but effective nonetheless. Another suggestion for understanding various mathematical concepts could be to study how and for what purpose the algorithms used most commonly were developed. If concepts are introduced when they are needed, and the student given some historical insight, it becomes easier for the student to remember the associated algorithm (Tobias, 1993).

Both Polya (1990) and Tobias (1993) expound on the use of estimation as an important and useful tool. Preoccupation with getting the "right answer" in their previous mathematics studies hinders many students' use of this extremely invaluable tool. Automatic usage of this tool would be beneficial to all students throughout their lives. The 1989 NCTM Standards call for, among other things, the teaching of paper-and-pencil estimation along with less computational drill and practice. "Real mathematics - the kind we need for everyday problem solving - involves estimation, at least for starters, so we can anticipate what the solution ought to look like before we punch numbers into our calculators" (Tobias, 1993, p.40).

Even educated guessing has its place in mathematics education. "Many a guess has turned out to be wrong, but nevertheless useful in leading to a better one" (Polya, 1990, p. 99). Wrong answers can be viewed as steps to obtaining the correct answer provided the student is willing to use the knowledge gained from the problem-solving process. As Tobias points out, "The process of checking one's guess very often mimics the algorithm or formula by which the problem will eventually be solved" (1993, p.144). Polya suggests learning from the problem by looking back. He admonishes students to check the result, check the argument, derive the result differently, use the result for some other problem, reinterpret the problem, interpret the result, or state the new problem. "A good teacher . . . brings in the scratch paper he used in working out the problem, to share with the class the many false starts he had to make before solving it" (Tobias, 1993, p.53).

Metacognition, self-talk, and overcoming mathematics anxiety. "Metacognitive skills are related to thinking about thinking, and more precisely, thinking about one's own learning. . . . The importance of spending effort on the development about thinking-about-thinking skills . . . becomes especially clear when it is realized that students who are able learners develop these skills intuitively" (Ganz & Ganz, 1990/1993, p. 64).

Self-interrogation is one important metacognitive technique. Brown, Bransford, Ferrara, and Campione (1982) suggest that successful learners use self-questioning among other strategies. Brophy suggests that teachers "model the strategic applications of skills via 'think aloud' demonstrations. These demonstrations make overt for students the usually covert strategic thinking that guides the use of the skills for problem solving" (1992/1994, p. 189). Along the same lines, Tobias suggests that the teacher should show the student the entire thought process used in solving problems (1993).

> "The best is, however, to help the student naturally. The teacher should put himself in the student's place, he should see the student's case, he should try to understand what is going on in the student's mind, and ask a question or indicate a step that *could have occurred to the student himself*" (Polya, 1990, p. 1).

Polya repeatedly suggests questions for the would-be problem solver to pose. Acquiring the skill to independently pose these questions is an underlying theme in his classic book, How to Solve It. Questioning oneself to solve a problem is one positive form of self-talk. According to Tobias (1993), replacing negative self-talk such as, "Oh, no, I can't do this problem!", with appropriately modeled questions, is a desired goal for remedial students. She also suggests using questioning self-talk when the student "goes blank".

Much mathematics anxiety is produced by giving students rules without understanding.

> "Math anxious people seem to have little or no faith in their own intuition. If an idea comes into their heads or a strategy appears to them in a flash, they will assume it is wrong. They do not trust their intuition. Either they remember the 'right formula' immediately, or they give up" (Tobias, 1993, p. 66).

Fear of making mistakes, in a seemingly arbitrary subject, leads to anxiety. Perhaps the greatest cause for anxiety, however, is the myth that mathematical ability is inborn, not the result of hard work. "Parents . . . unwittingly foster the idea that a mathematical mind is something one either has or does not have" (Tobias, 1993, p.53).

> "Only in the United States do people believe that learning mathematics depends on special ability. In other countries, students, parents, and teachers all expect that most students can master mathematics if only they work hard enough" (National Research Council, 1989, p.10).

To reduce anxiety, it is important to dispel the myth that mathematical ability is inborn. Also, students need to reject the ideology that "if we haven't learned something so far, it is probably because we can't" (Tobias, 1993, p.62). Remedial mathematics programs should include, along with the necessary instructional help, strategies to reduce mathematical anxiety by substituting positive beliefs for these negative ones (Dew, Galassi, & Gallasi, 1984). Because test anxiety and mathematics anxiety seem to overlap, as Sarason (1987) notes, in general, strategies used for mathematics anxiety will work for test anxiety as well (Dew, Galassi, & Gallasi, 1984).

Clarity and language. Conciseness is an important component of clarity. Clear explanations and modeling from the teacher are important, but so are opportunities to answer questions about the content (Brophy,1992/1994). "The ability to 'say what you mean and mean what you say' should be one of the outcomes of good mathematics teaching. This ability develops as a result of opportunities to talk about mathematics, to explain and discuss results which have been obtained, . . ." (Cockroft, 1982, p. 72).

"Differences in meaning between common language and mathematical language do get in our way . . . . There are conflicts between the common everyday use of words and the use of words in math" (Tobias, 1993, p. 37). Many words, like "multiply", mean one thing when first introduced in the language context, but in the mathematical context may mean something quite different. Additional confusion arises when "multiply" is used to represent the mathematical operation, because it has two different effects which depend on the specific numbers involved. Tobias (1993) gives many more examples of words eliciting this kind of confusion and symbols with ambiguous meanings. In her book, she quotes H. Poincare, a mathematician and educator, as saying, " . . . a definition is satisfactory only if the students understand it" (1993, p. 54).

Humor, games, and cooperative learning. Remedial adult students can benefit from the use of humor in the curriculum. Retention and comprehension are both aided by the use of humor (Kaplan & Roscoe, 1977). Tobias (1993) readily uses cartoons to illustrate many situations in her book, Overcoming Math Anxiety, and Polya uses humor throughout his book, How to Solve It.

"Given the choice between two techniques, choose the one involving the learners in the most active participation" (Knowles, 1980, p. 240). Silberman (1990) also stresses the principle of active learner participation. The use of games and manipulatives in the classroom provides for active student participation.

A relaxed and collaborative climate for review is preferable to the competitive and judgmental climate needed in pedagogical situations (Knowles, 1980). Tobias encourages teachers to have students work in groups, recognizing that competition increases tension (1993). Schoenfeld (1985) suggests that working in small groups facilitates the learning process. For example, as students justify to other group members their reasons for choosing alternative solutions, articulation of knowledge and reasoning is promoted. Students also receive practice in collaboration, a skill needed in real-life problem solving. "The growing body of research on cooperative learning indicates that this mode of instruction is an effective instructional technique with students of all ability levels and in all areas of mathematics, from remedial mathematics to college calculus and beyond" (Prichard & Bingaman, 1993, p. 221). Slaven (1990), a staunch proponent of cooperative learning strategies, proposes that these strategies have a positive effect on student self-esteem, thereby reducing mathematics anxiety.

CHAPTER 3

Procedures

Introduction

This chapter presents a discussion of the procedures used in the development and limited evaluation of a basic skills mathematics curriculum for adults. A detailed discussion of the development of the curriculum is presented first, followed by discussions of each of the following: justification of curriculum changes, evaluation of student progress, and a survey of instructors' opinions.

Development of the Curriculum

This basic skills mathematics curriculum was developed to provide an alternative means to increase adult learners' proficiency in basic mathematics. This was accomplished by changing the sequencing of the contents, as well as the actual learning experiences, to be more effective in helping the adult learners integrate mathematical concepts with their life experiences. More specifically, this was developed to replace the traditionally sequenced MAT-010 curriculum previously in use at Gloucester County College.

This curriculum was developed using Gagne's model of curriculum development (Appendix A). His model was selected because it best suited this revision of an already existing curriculum and allowed for the formative and summative evaluations that are needed to assess the curriculum while being implemented. Because the researchers were changing the sequencing of this particular curriculum, Gagne's philosophy of hierarchical sequence seemed appropriate as a guide (1987, p. 231). Another reason for choosing his model was that it is goal driven, a real necessity for this particular curriculum. The model was followed in its entirety. The following is a description of how each step in his model was accomplished.

Analysis of needs and identification of needs. Prior to the development of this curriculum, the basic skills mathematics classes at Gloucester County College experienced problems with retention, attendance, and successful completion. These problems indicated that the curriculum was not meeting the needs of the students. All students are required to pass a basic skills placement test to continue towards their degree. Passing this test is a requirement before they can obtain credit for, or even enroll in, any mathematics course at the college. The main purpose, then, of the MAT-010 course is to prepare the student for successful completion of this test. The students are mostly "adult learners" who have already completed many courses throughout their lives that dealt with the basic skills mathematics topics, but have had trouble either with the retention of these skills or with the test format itself. The previous curriculum did not take into account the special needs and restraints of the adult learners nor draw on the varied experiences that they all have acquired.

Goals and objectives. The goals of this curriculum were more or less dictated by the college. It should be noted that passing the basic skills placement test is the main reason for taking this course and as such is a goal of each individual student. Another important goal is the development of thinking, reasoning, and problem solving skills.

Identify alternative ways of meeting the needs. After spending hours in search of alternative learning experiences already being used and inspecting every publisher's texts at the NJEA Convention, the researchers determined that there exists very little material for adult learners. The researchers also asked colleagues, GED mathematics teachers, and anyone remotely connected with this field for input and ideas. One professor at Gloucester County College, Roseann Foglio, gave very meaningful input and was very interested in this project and its implementation.

Manipulatives, study groups, collaborative learning, educational games, and competitions were identified as alternative methods of instruction to supplement the already exiting Academic Support Lab and Computer Lab. Adult remedial mathematics students have all been taught this material many, many times before using mainly the methods of lecture and drill. For this reason, this curriculum strives to limit the use of these two types of learning activities.

Design of system components. Most of the system components were already in place. The MAT-010 course is a fifteen-week course consisting of two classes per week which meet for one hour and fifteen minutes each. A placement test is given to each student prior to enrollment in the class. This is used as a preassessment test. Additional individual learning takes place in the Computer Lab and the Academic Support Lab where tutoring is provided. Components that were designed specifically for this new curriculum were based on the researchers' application of related literature. They include learning experiences that are composed of both group-oriented and self-paced activities such as games, competitions, and collaborative assignments.

The course is divided into the following four units: (1) rational numbers, (2) approximations, estimations, and test taking, (3) multiplication and division of rational numbers, and (4) addition and subtraction of rational numbers. Each unit includes instruction, applications to real life, and games. At the end of each unit, a comprehensive multiple choice test similar to the placement test is given to assess how close each student is to the goal.

For the structure of this curriculum to be effective, the researchers agree with Jerome Bruner and Robert Gagne that the structure is inherent and needs to be revealed to the learner in deliberate, well thought out stages (Deighton, 1971).

<u>Analysis of resources required, resources available, and constraints</u>. This curriculum requires adult learning activities and, specifically, adult manipulatives and games. These are not available commercially.

The Academic Support Lab is already in existence and provides tutoring and computer availability among other features. Tests similar to the placement test have been devised.

One constraint is getting a group of instructors together on their own time to facilitate implementation of this new curriculum. Because this curriculum is so different from the previous one, problems are encountered with personnel who just don't want to change. This curriculum must be implemented in its entirety to be both effective and meaningful.

Action to remove or modify constraints. The original three pilot classes were taught by three instructors who were enthusiastic and trained to use the materials and text. The current field testing consists of all of the MAT-010 instructors and all of the classes. There are scheduled semi-monthly meetings to address specific problems or concerns, promote enthusiasm, discuss student feedback, and facilitate summative evaluation. These meetings are collaborative in nature. Statistics are being gathered to be presented to future instructors at orientation. It would be imperative that the instructors using the curriculum be given training specific to it.

Selection or development of instructional materials. Because of the unavailability of materials for this project, most of the researchers' time was spent devising original materials. While devising these materials, they needed to keep in mind the cumulative nature of learning. It was necessary to devise each activity so that it draws on a previously learned experience as was expounded by Gagne (Deighton, 1971). After researching all available related literature, the applicable theories were incorporated into this new curriculum.

Design of student assessment procedures. In order to pass this course, the college has mandated that a student pass the placement test. Formative testing ensures the preparedness of each student. At the end of each unit, students are given a comprehensive test similar to the placement test. These tests are used diagnostically to indicate student progress and as a tool in developing test taking strategies.

Field testing. The researchers and one other instructor taught pilot classes using the new curriculum. Roseann Foglio observed classes and helped with necessary revisions to make the course more effective. Feedback from these faculty members and from the students involved helped with initial formative evaluation. Additional formative evaluation and teacher training will be ongoing.

Adjustments, revisions, and further evaluation. It is intended that adjustments and revisions will be made after a full semester using this curriculum.

Summative evaluation. This should take place only after several semesters have elapsed. Hopefully, the information collected during this period will show the curriculum to be more effective. This would be shown by increases in student retention and increases in the percentage of students who succeed in accomplishing the requirements to go on in their mathematics endeavors. At that time, questionnaires completed by participating faculty would be used to evaluate and revise the curriculum.

Operational installment. Every MAT-010 class is using the new curriculum and text for three semesters concluding in the spring of 1997. The curriculum is still in its field-testing stage. Formal operational installment will occur only after favorable summative evaluation.

Justification of Curriculum Changes

This new curriculum addresses the problems of the traditional curriculum. In the process of its development, each curriculum change was justified. The curriculum applies concepts gathered from related literature which were cited to substantiate each change. These concepts, coupled with the researchers' ideas for a new sequence of topics, were the foundation for the development of this new curriculum.

Evaluation of Student Progress

A limited evaluation of students' progress using the new basic skills mathematics curriculum was conducted at Gloucester County College over a seven week period from January to March of 1996. All of the nine MAT-010 classes used the new curriculum during the spring 1996 semester. Five classes were given a version of the New Jersey College Basic Skills Placement Test as a pretest. This test is an indicator of the students' computational skills with all forms of rational numbers. Two of the four units of the new curriculum were covered in all of the classes. These units only covered the relationships of rational numbers and the use of approximation skills, problem solving skills, and test taking strategies. No computational skills were taught prior to the mid-semester test. The five classes were given a second version of the New Jersey College Basic Skills Placement

Test as the mid-semester test. A dependent t test was used to determine whether the first two units had affected student achievement in computational skills.

Survey of the Instructors' Opinions

All MAT-010 faculty were surveyed at the end of the seven week period when they had completed the first two units. All of the instructors had previously used the traditional sequence to teach basic skills mathematics. This survey evaluated the sequencing of the first two units and the attitudes of the students by means of an opinionnaire. The survey used was validated by the jury method. It was also field tested prior to distribution.

CHAPTER 4

Analysis of Data

Introduction

This chapter presents the analysis of the development and limited evaluation of a basic skills mathematics curriculum for adults. The limited evaluation of the new curriculum is followed by the results of an evaluation of student progress and a survey of instructors' opinions.

Evaluation of the Curriculum

The limited evaluation of the new curriculum consists only of citing text locations of selected examples addressing each of the concepts gathered from the related literature. The evaluation is summarized in Table 1.

Table 1

Evaluation of the Curriculum

Concept Addressed	Selected Examples
Connectedness and teaching for understanding	"Links" - pages 5, 6, 19, 31, 33, 49, 69, 109 Real-life applications - pages 78-80, 155-160 Using holistic geometric interpretation to understand square roots - pages 69-74

Table 1, continued

Concept Addressed	Selected Examples
Sequencing content	Table of Contents - page vii Preface - page ix Percent problems addressed - all four units
Distributed practice	"Mental Math Challenge 1, 2, and 3"- pages 17, 57, 67
Use of alternate methods and estimation to teach problem solving	Alternative methods instruction - pages 11, 20, 35, 63, 125 Estimation - Unit 2
Metacognition, self-talk, and overcoming mathematics anxiety	Letters to the student - pages 1-2, 9-10, 90 Modeling self-talk - pages 27, 37-38, 42, 96, 114, 133 Preface - pages ix-x
Clarity and language	Illustration - page 105 Language of percent - page 123
Humor, games, and cooperative learning	"Irv" - pages 2, 22, 74 Cartoons and graphics - throughout text Card games, puzzles, brain teasers - pages 163-179

Evaluation of Student Progress

Using pretest scores and posttest scores of the New Jersey College Basic Skills Placement Tests for fifty-eight students, a two-tailed dependent t test was performed. The test indicated that the results were significant at the .01 level. The results are shown in Table 2.

Table 2

Evaluation of Student Progress

Number in Sample (n)	58
Mean of the Differences of Scores $(\bar{d})$	29.1897
Standard Deviation of the Sample (S_d)	17.8005
Mean of the Difference of the Population $(\bar{u})$	0
t-score	12.4886 *

* Significant at the .01 level

Analysis of the Survey

The survey was comprised of twelve questions (Appendix B). It contained both forced-response and open-ended questions. The results of the forced-response questions indicated that the new sequencing, the teaching of estimation skills, and testing in class with review appeared to be beneficial to the students. The students seemed to be more attentive, participated more, and responded favorably to the distributed practice activities. These results are summarized in Table 3.

Table 3

Responses to Opinionnaire

Item No.	Strongly Agree	Agree	Undecided	Disagree	Strongly Disagree
1	3	1	1	0	0
2	0	0	0	5	0
3	1	3	1	0	0
4	3	1	1	0	0
5	0	5	0	0	0
6	3	1	1	0	0
7	0	0	1	2	2
8	0	5	0	0	0
9	4	1	0	0	0
10	3	1	0	1	0

The results of the open-ended questions follow.

It was noted by one respondent that the use of an approach of understanding concepts versus learning a particular method fosters students' confidence. The respondent also mentioned that estimation was instrumental in building positive test-taking and study habits. Another respondent commented that the information is presented in such a way that the students realize their basic mathematics inadequacies. The introduction of rational number relationships early in the text enables the students to understand number relationships without the complication of arithmetic operations.

Two respondents identified the need for inclusion of mixed review activities in Unit 3. Additionally, one respondent suggested the inclusion of supplemental quizzes. One final comment was that this curriculum is such a relief from the drudgery of tedious review of arithmetic operations, thus proving that one can elevate as one remediates.

CHAPTER 5

Summary, Conclusions, and Recommendations

Introduction

Chapter five concludes the development and evaluation of a new basic skills mathematics curriculum for adult learners. The first section, Summary of the Findings, provides a synopsis of the study. The second section, Conclusions, states the conclusions drawn by the authors as a result of this study. The final section, Recommendations for Further Study, suggests further research for future analysis of the curriculum.

Summary of the Findings

The limited evaluation of the new curriculum concluded that each of the concepts gathered from the related literature was addressed by the new curriculum.

Student progress was evaluated using pretest scores and posttest scores of the New Jersey College Basic Skills Placement Tests for fifty-eight students. A two-tailed dependent t test showed that the improvement in computational skills evidenced by the achievement test scores was probably not attributable to sample error. The difference in scores was significant at the .01 level.

The results of the forced-response questions of the opinionnaire showed that the instructors' opinions towards the new curriculum were favorable. This was determined by the use of the Likert Method.

Conclusions

The new curriculum successfully addressed each of the concepts gathered from the related literature. Using a dependent t test, results indicated that the students' computational achievement was significantly improved after covering the first two units of the text, which did not include computational instruction. The difference was significant at the .01 level. These results concur with the NCTM Standards that concluded that remediation is more effectively taught by methods which stress understanding and not computational drill.

The survey of instructors showed not only were the instructors in agreement with the new curriculum, but none of the responses showed any disagreement. The majority of instructors strongly agreed with the new sequencing, testing in class, and teaching estimation. Also, the majority noted an increase in class participation and attentiveness, and that the students seem to like the new sequencing and the card games.

Recommendations for Further Study

One recommendation is that the instructors be surveyed again at the conclusion of the semester. This would improve the reliability of the study. The researchers also recommend that the evaluation of student progress be repeated at the end of the semester, on a greater scale, and on a regular basis in future semesters.

Another area that the researchers feel should be investigated is the impact that the successful completion of the first two units has on the mathematics anxiety level in remedial adult learners. Because unit two deals extensively with coping with test and mathematics anxiety, ideally there should be a reduction in anxiety along with the improved achievement already evidenced.

The researchers would like to see this curriculum tested in other community colleges and research done on its effectiveness in contrast to existing remedial mathematics curricula. The curriculum could also be studied for possible use in other appropriate andragogical settings, replacing the traditional curriculum that emphasizes computational drill. GED classes and high school remedial mathematics classes are two examples of where this new sequencing may prove beneficial. In addition, adoption of a core curriculum, as suggested by the NCTM, necessitates research that addresses the mathematics education of underachieving students (1993).

REFERENCES

Brophy, J. (1994). Probing the subtleties of subject-matter teaching. In F. Schultz (Ed.), Annual editions: Education 94/95, (pp. 188-191). Guilford, CT: The Dushkin Publishing Group. (Reprinted from Educational Leadership, April 1992, 49 (7), 4-8)

Brown, A., Bransford, J., Ferrara, R., & Campione, J. (1982). Learning, remembering, and understanding. Technical Report, 244 (June), 29-32.

Bruner, J. S. (1977). The process of education. Cambridge, MA: Harvard University Press.

Cockcroft, W. (1982). Mathematics counts: Report of the Committee of Inquiry into the teaching of mathematics in schools. London: Her Majesty's Stationery Office.

Deighton, L. (1971). Textbooks: Role in education. In L. Deighton (Ed.), The Encyclopedia of Education (Vol. 9), (pp. 212-213). New York: Crowell-Collier Education Corporation.

Dempster, F. (1994). Exposing our children to less should help them learn more. In F. Schultz (Ed.), Annual editions: Education 94/95, (pp. 202-205). Guilford, CT: The Dushkin Publishing Group. (Reprinted from Phi Delta Kappan, February 1993, 433-437)

Dew, K., Galassi, J., & Galassi, M. (1984). Math anxiety: Relation with situational test anxiety, performance, physiological arousal, and math avoidance behavior. Journal of Counseling Psychology, 31, 580-583.

Dewey, J. (1938). Experience and education. New York: Collier-Macmillan.

Friedlander, J. (1979, August). Developmental mathematics. Junior College Resource Review, 3-5.

Gagne, R. M. (1979). Principles of instructional design. New York: Holt, Rinehart, and Winston.

Gagne, R. M. (Ed.). (1987). Instructional technology: Foundations. Hillside, NJ: Lawrence Erlbaum Associates.

Ganz, B.C., & Ganz, M. N. (1993). Linking metacognition to classroom success. In K. Cauley, F. Linder, & J. McMillan (Eds.), Annual editions: Educational Psychology 93/94, (pp. 64-67). Guilford, CT: The Dushkin Publishing Group. (Reprinted from The High School Journal, February/March 1990, 180-185)

Houle, C. O. (1972). The design of education. San Francisco: Jossey-Bass.

Knowles, M. (1980). The modern practice of adult education. Chicago: Follette.

National Council of Teachers of Mathematics. (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: The Council.

National Research Council. (1989). Everybody counts: A report to the nation on the future of mathematics education. Washington, DC: National Academy Press.

Newmann, F. M., & Wehlage, G. G. (1994). Five standards of authentic instruction. In F. Schultz (Ed.), Annual editions: Education 94/95, (pp. 206-209). Guilford, CT: The Dushkin Publishing Group. (Reprinted from Educational Leadership, April 1993, 50, (7), 8-12)

Oliver, A. I. (1965). Curriculum improvement. New York: Dodd, Mead, & Company.

Pate, P. E., McGinnis, K., & Homestead, E. (1994). Creating coherence through curriculum integration. In J. Beane (Ed.), Toward a Coherent Curriculum, (pp. 62-70.) Alexandria, VA: Association for Supervision and Curriculum Development.

Polya, G. (1990). How to solve it. London: Penguin Books.

Preston, T. (1995, winter). Adult education in Girl Scouting. Girl Scout Leader, 26.

Prichard, M. & Bingaman, S. (1993). Instructional activities and decisions. In P. Wilson (Ed.), Research ideas for the classroom: High school mathematics (pp. 217-231). New York: Macmillan Publishing Company.

Sabatino, D. A., & Mann, L. (1982). A handbook of diagnostic and prescriptive teaching. Rockville, Maryland: An Aspen Publication.

Saranson, I. (1987). Test anxiety, cognitive interference, and performance. In R. Snow and M. Farr (Eds.), Aptitude, learning, and instruction: Volume 3. Conative and affective process analyses (pp. 131-142). Hillsdale, NJ: Erlbaum.

Schoenfeld, A. (1985). Mathematical problem solving. Orlando: Academic Press.

Silberman, M. (1990). Active training. Lexington, MA: Lexington Books.

Slavin, R. (1990). Cooperative learning: Theory, research, and practice. Englewood Cliffs, NJ: Prentice-Hall.

Tobias, S. (1993). Overcoming math anxiety. New York: W. W. Norton & Company.

Walberg, H. J. (1988, March). Synthesis of research on time and learning. Educational Leadership, 45, 76-85.

APPENDIX A

Gagne's "Steps in Instructional System Development"

1. Analysis and identification of needs
2. Definition of goals and objectives
3. Identification of alternative ways to meet needs
4. Design of system components
5. Analysis of:
   (a) Resources required
   (b) Resources available
   (c) Constraints
6. Action to remove or modify constraints
7. Selection or development of instructional material
8. Design of student assessment procedures
9. Field testing: formative evaluation and teacher training
10. Adjustments, revisions, and further evaluations
11. Summative evaluation
12. Operational installment

APPENDIX B

Opinionnaire for Instructors

SURVEY

Part A - The following statements represent opinions. Please check your position on the scale as follows:

1 - I strongly agree. 2 - I agree. 3 - I am undecided.
4 - I disagree. 5 - I strongly disagree.

1.) The sequence of this new curriculum appears to be more beneficial to the students.

1_____ 2_____ 3_____ 4_____ 5_____

2.) The students seem to dislike this new sequence.

1_____ 2_____ 3_____ 4_____ 5_____

3.) The students seem to like the activities involving the decks of cards.

1_____ 2_____ 3_____ 4_____ 5_____

4.) It is more enjoyable teaching this course using the new curriculum.

1_____ 2_____ 3_____ 4_____ 5_____

5.) The students seem to be more attentive and/or interested in the material being taught.

1_____ 2_____ 3_____ 4_____ 5_____

6.) Testing in class and going over the problems after completion is a beneficial learning experience.

1_____ 2_____ 3_____ 4_____ 5_____

7.) Comprehensive tests are less beneficial than unit tests.

1_____ 2_____ 3_____ 4_____ 5_____

8.) The students seemed to respond favorably to the 'Mental Math Challenges' spaced throughout Units 1 and 2.

1_____ 2_____ 3_____ 4_____ 5_____

9.) Teaching estimation as an essential learning tool has benefitted your students this semester.

1_____ 2_____ 3_____ 4_____ 5_____

10.) An increase in class participation has been noted.

1_____ 2_____ 3_____ 4_____ 5_____

Part B - Please complete each of the following.

11.) List any advantages/strengths pertaining to this curriculum.

12.) List any disadvantages/weaknesses pertaining to this curriculum.

13.) Comments (particularly those comparing this new curriculum to the old curriculum):

APPENDIX C

Text

ReVIEWing MATH

A Closer Look for Better Understanding

Roseann Foglio
Shirley Hofer
Annette M. Schenkel
Gloucester County College

McGraw-Hill, Inc.
College Custom Series

New York St. Louis San Francisco Auckland Bogotá Caracas Lisbon London Madrid Mexico Milan Montreal New Delhi Paris San Juan Singapore Sydney Tokyo Toronto

Reviewing MATH

A Closer Look for Better Understanding

1 2 3 4 5 6 7 8 9 0 SEM SEM 9 0 9 8 7 6 5

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and Frank Foglio for the many copies of each phase of the manuscript.

Instructions

How to Use this Textbook

Notice the red rectangular areas throughout this book. These areas contain answers to the problems or questions. Do each problem as you proceed, writing your answer on your paper. **Immediately** check yourself by placing the RED ACETATE CARD over the red rectangular area to see if you are correct. If you have made a mistake, draw a circle around your answer. This will prepare you to ask pertinent questions. It will also help indicate those areas on which you need to concentrate.

How to View Stereograms (on the cover)

Learning to see these images may take some practice. If you don't get them at first, don't be discouraged. With a little practice it usually becomes much easier to see them.

The viewing environment is very important. For new viewers, it should be relaxed and quiet with good lighting. Be sure to keep the image still and level at all times. You need to be able to relax and concentrate for a while without disruptions.

Method 1: Position the image a comfortable distance away from you - usually 18 to 24 inches. Allow your eyes to relax and 'space out' or wander away from a fixed focal point. When you see the repetitions in the pattern, try to 'lock in' on them so that they overlap. You should begin to see the 3-D image emerge. Don't force it too hard, but slowly try to bring it into focus. When you have it, you should be able to look around at other parts of the picture.

Method 2: Look at the "convergence dots" at the top of the picture and allow your eyes to relax and cross slightly until you see three dots. Look at the center dot and wait until your eyes have focused comfortably on it. Slowly lower your view to the rest of the image, and you should see the 3-D image.

Method 3: Position the image so that it is touching your nose. You won't be able to focus on it, but that's OK. Relax your eyes so that you see a blurry mess of colors, and wait a few seconds until it feels comfortable. Now, slowly move the image away from your face. Don't try to look at the image or focus on it, just relax and slowly move it away. Soon, you should start to see depth in it. Let the image develop, but don't force it. Be patient, and soon the 3-D picture will become clear. If you lose it at any time, just start over.

Solution to stereogram on the cover: M A T H

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This book is *not* a book of tricks. This book *is* a unique, non-threatening, and proven approach to mastering essential math. Math educators are always seeking better, more effective methods for helping math anxious adults. The approach offered presents a new method of learning. Instead of following the traditional sequence of math concepts, the students are shown how numbers actually work.

The fallacies that especially the American people labor under, *do* hinder our mathematical achievement as a whole. Americans tend to think that people who excel in math, or science for that matter, must be gifted. The Japanese, in contrast, consider people who excel to be those who have worked very hard at the particular subject. When a student has any difficulty with math, he is made to feel as if he is not "gifted" and rather than work hard to master the math skill, uses his "not- giftedness" as an excuse for continued failure. Unfortunately, the people around him, products of the same system, most times do not help the student to see this fallacy, as quite a few of them are still suffering from the same misconception about themselves. Sheila Tobias in Overcoming Math Anxiety substantiates this fact about American attitudes on math achievement with many documented studies and shows, additionally, how this fallacy extends particularly to the supposed gender differences in math abilities.

There are really no mathematically illiterate people, only those who have not learned to do math in a way that works for them. Unfortunately, the key to eventually becoming math literate is for the student to be presented with the materials in *different* ways and not merely the same way over and over again. Students need to be shown that there is more than one way to solve every math problem. Not only are students given the impression that there is only one answer to a problem, they are also told, to varying degrees, that there is a "right way" to do a math problem. In many schools, students are only taught one method for solving any given problem. Even worse, should they stumble upon their own equally correct algorithm, the student is usually reprimanded. Knowles Dougherty, a specialist in teaching the math-disabled, demonstrated the adult learner's problem with this kind of restrictive math background. The adults in the study were all asked to do a word problem in their heads and tell how they did it, without giving the answer. It turned out, according to the study, that each adult was ashamed of the system (or algorithm) they used to solve the problem. They assumed that because there was only one correct answer to any given math problem, there was only one "right way" to do the problem and it was not their way. Students need to be encouraged to use personal reference points and intuition to restructure problems so that they make sense. From the earliest of grades, intuition is discouraged and using a student's knowledge of his own world is almost never tapped as a resource.

As for "showing how numbers actually work," consider the following directions from a traditional textbook:

"Just express the hundredth as a decimal, delete the decimal point, and place the percent sign after it." This is exactly the type of math instruction that makes for anxious, math illiterates. A person who remembered how to do this particular problem from prior knowledge is the only one who could do what the author is instructing. The person having trouble with math, will be asking how to express the hundredth as a decimal, and why delete the decimal point, etc. The person experiencing math anxiety will not remember this kind of algorithm accurately, if at all. Usually, what happens is that the student remembers that the decimal point must be moved, but which way and how many places, is elusive. Understanding a problem, on the other hand, would eventually lead a student to come up with the algorithm on his own. *Math anxious students do not need more rules to memorize*, they need fewer rules and more understanding. If a person can connect something to the long term memory, it will be much easier to retrieve the information. To what long term information could the above example be connected? Life experiences can be used to develop methods of doing math problems.

A suggestion for understanding various math concepts could be to study how and for what purpose the algorithms used most commonly were developed. If concepts are introduced when they are needed, and the student given some historical insight, it becomes easier for the student to remember the associated algorithms. Many students, according to many learning style specialists, can learn best when first shown how the material fits into the much bigger picture.

Teachers sometimes do their students a very big disservice by portraying themselves as infallible, always able to come up with the correct answer easily and without any error along the way, and appearing sometimes to pull the answer right out of the air. If a student does not understand how the teacher obtained the correct result, it sometimes leads to the false belief that the student is incapable of ever solving these types of problems. This book encourages and leads the student to make use of self-talk, a very useful learning tool which helps to overcome this fear.

This book employs methods to bolster the esteem and expectations of the readers. It encourages the student to reason, using real life experiences rather than depending on one's ability to memorize. The student becomes "free" to develop understanding of the subject.

Letter to the Student

Dear Student,

It seems that many people who have been taught arithmetic don't feel confident when they need to apply some basic concepts. There are probably an overwhelming number of reasons for this dilemma. However, whatever the reason, what can be done now? No one who seriously wants to continue his or her education should be stopped by an obstacle that can be, in most cases, removed.

The material presented in this book is no different from that in any basic math book, but the approach is very different. Have you seen the three-dimensional computer art that is popular now? You stand so close that the images are a blur. Then as you move back, looking at it from a different angle, another image pops out all of a sudden. Maybe looking at the same old math from another angle will help to make it more clear. Just as with the computer art where some people see the images immediately while it takes others a little longer, so too may be the case with the basic math in this book. Stay focused. Be patient. It will be worth the effort!

Please react to the following statements and check those that describe how you feel about math.

____1. I'll never be able to do math because I don't have a mathematical mind.

____2. I get all the answers in the exercises right, but I flunk the tests when they are mixed up.

____3. I'll never use this math.

____4. Word problems make me sick.

____5. Why can't I just use my calculator?

____6. My answers are usually right but the decimal points are in the wrong spots.

____7. It's been years since I've worked with fractions.

____8. The teacher confuses me.

____9. I thought I knew how to read until I opened my math book!

____10. I never know when to use the percent sign.

If you checked any of the above statements, **THIS BOOK'S FOR YOU!**

This book will answer the following questions and **many, many more**!!

*** When is using a fraction a better choice than using a decimal?

*** Which way do you move that %@#&! decimal point?

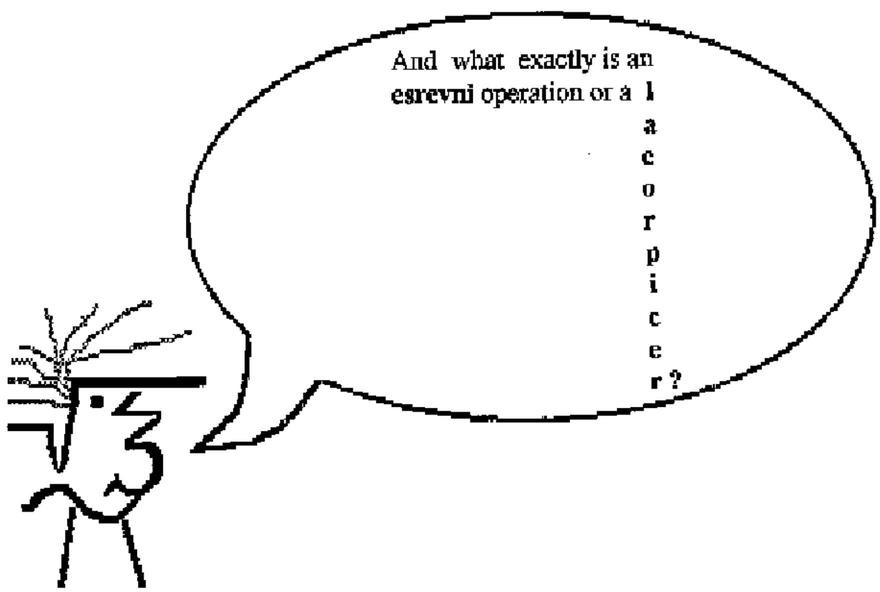
*** How many places do you move that %@#&! point?

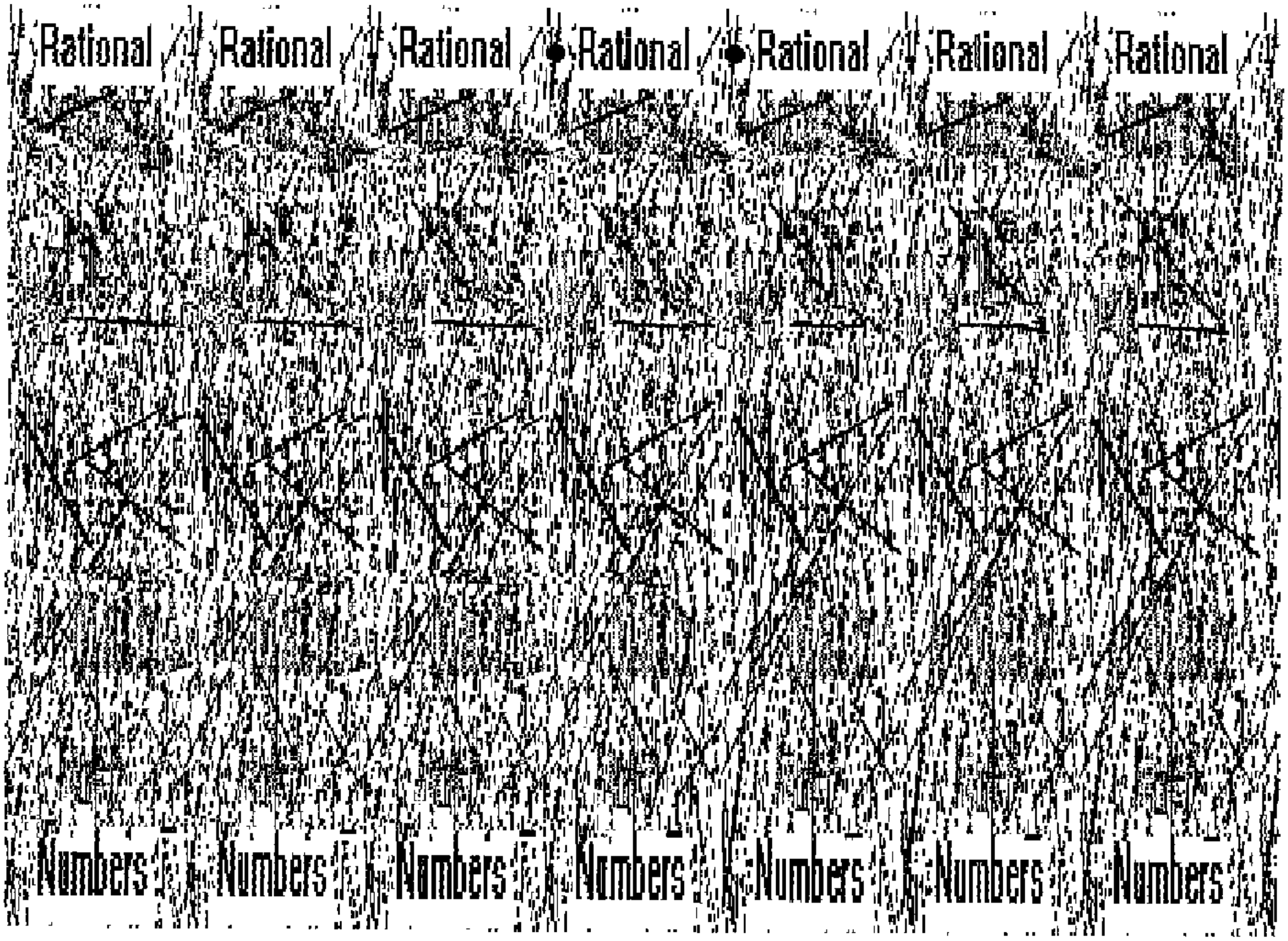
*** Do you divide the bottom number into the top or vice-versa?

*** Which fraction do I flip? And when?

*** Can I just get rid of the percent sign?

*** How come I need to work with a fraction and a percent at the same time? And what exactly does $\frac{1}{2}$% mean?




Rational
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Unit 1

1.1 Introduction

Although other sets of numbers are used in all branches of mathematics, our discussion will focus on the set of rational numbers. Many rational numbers are elements in more than one set. Different sets of numbers are useful in certain types of applications. Therefore, it is important and convenient to know how they compare with each other.

Those ideas, procedures, or definitions that join and connect the topics previously learned to those currently being presented will be called links: .

Quotient - the result of dividing one rational number by another

Example: $\frac{8}{4} = 2$, where 2 is the quotient

Example: $6 \div 7 = \frac{6}{7}$, where $\frac{6}{7}$ is the quotient

Integer - a positive or negative quotient that can be expressed as a whole number divided by one

Example: $+\frac{8}{1} = +8$, where $+8$ is an integer

Example: $-\frac{5}{1} = -5$, where -5 is an integer

Terminating decimal - a decimal fraction whose denominator can be identified

Example: $\frac{5}{8} = 8\overline{)5.000}$ with quotient $.625$, where $.625$ is a terminating decimal

Repeating decimal - a decimal in which a digit or block of digits repeats on to infinity

Example: $.123123123\overline{123}$

Rational number - a number that can be expressed as the quotient of two integers. These are mostly all of the numbers that we work with daily.

All fractions are *rational* numbers.

Examples: $\frac{1}{5}$, $\frac{72}{87}$, $\frac{287}{1000}$, $\frac{16}{5}$

Mixed numbers are *rational* numbers. Change them to improper fractions.

Examples: $2\frac{1}{7} = \frac{15}{7}$, $3\frac{4}{9} = \frac{31}{9}$

Whole numbers are *rational* numbers. Write them as whole numbers divided by 1, with the whole number as the numerator and 1 as the denominator.

Examples: $7 = \frac{7}{1}$, $3 = \frac{13}{1}$, $0 = \frac{0}{1}$

Terminating and repeating decimals are *rational* numbers. Some even refer to decimals as decimal fractions. We actually *read* decimals as fractions. For example, 0.12 would be read as "twelve hundredths" the same as $\frac{12}{100}$ would be read.

Examples: $0.12 = \frac{12}{100}$, $1.7 = 1\frac{7}{10} = \frac{17}{10}$, $0.\overline{3} = \frac{1}{3}$

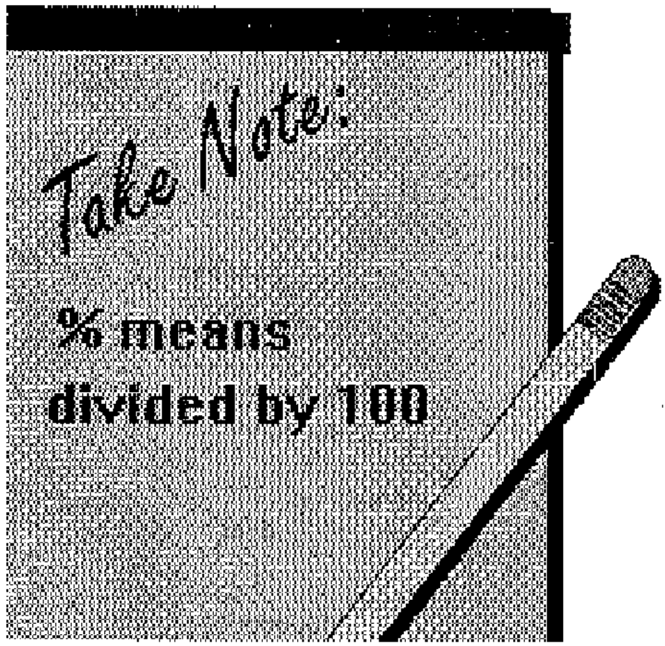
Here is an example showing that repeating decimals are rational.

To show that $0.\overline{12} = \frac{4}{33}$,

$$
\begin{aligned}
\text{Let } N &= .\overline{12} \\
100N &= 12.\overline{12} && \text{(multiply both sides by 100)} \\
-N &= -.\overline{12} && \text{(subtract N from both sides)} \\
\text{so, } 99N &= 12 \\
N &= \tfrac{12}{99} = \tfrac{4}{33} && \text{(divide both sides by 99 and reduce)}
\end{aligned}
$$

Percents are *rational* numbers. Percent means per hundred, or out of 100, or divided by 100.

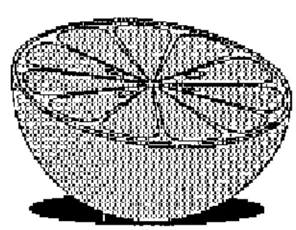
Example: 7% means 7 out of 100, or $\frac{7}{100}$, or $7 \div 100$ (7 divided by 100).



$$= \div 100$$

$$= \frac{\quad}{100}$$

We should study these types of rational numbers together because they follow similar rules of operations and are interchangeable. It is very necessary to understand all numbers as a whole, not only types or groups of numbers.



Notice this grapefruit half. There are different ways to refer to the quantity of grapefruit as pictured:

$\frac{1}{2}$ of a grapefruit   0.5 of a grapefruit   50% of a grapefruit

Each of these describes the picture.

Dear Reader,

"Self-talk" is anything one says to oneself. It can be positive, negative, encouraging, discouraging, uplifting, self-defeating, productive, or counter-productive. Everyone does some kind of "self-talk". Most students need to be shown how to make their self-talk more positive, motivating, and answer-seeking.

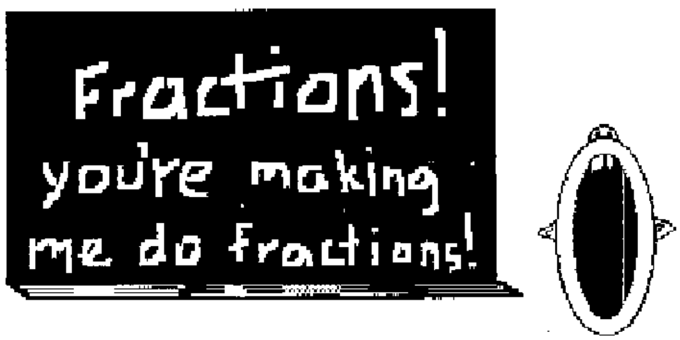
One area of self-talk that we will be addressing in Unit 2 is what we say to ourselves in a test situation. It is not only important to know the math concepts on a test, but also to free the mind to demonstrate what we know.

The second crucial area for self-talk is problem solving. When we truly learn mathematical concepts we begin to see how the ideas and methods "fit" together. Like a jigsaw puzzle, there is a sense of accomplishment as we see each piece, properly placed, helping to achieve the final result. We know that for a rectangular puzzle, we will have four corners and straight edges framing the puzzle. This is how we begin to think or use positive and productive "self-talk" to complete the task.

"Self-talk" also involves thinking about how to use the knowledge we already have to solve a new problem or application. It is a technique to practice so that a logical, selective approach to problem solving, concept connection, and real learning can occur. When we ask ourselves questions about how to begin to solve a problem, what result we are trying to achieve, what information is given and how does it "fit" into the solution we are using, we will be using "self-talk". Sometimes, "self-talk" is used to reinforce learning new concepts by reviewing. Most often, however, it will be the label used to describe the unique way each person tries one step after another in a solution, and then accepts or rejects that route to the solution, continuously proceeding toward the goal or answer in a problem. Each problem, therefore, does not depend solely on memorization of rules but on how concepts are connected and fitted together for a solution.

Another comparison might be to searching a destination using radar. The path taken by radar is a zigzag toward the goal. When the route goes too far to the left, it corrects and goes to the right. Then, as it gets too far to the right it changes direction again until it accurately meets its mark. Even wrong answers can often lead to the right answer, when viewed as detours rather than a dead end. Too often in mathematics, it is believed that only one route is best and it should be understood that how a person reaches a conclusion or answer may be unique and creative. The significant part of an *application is that the goal is achieved in a timely fashion.*

Throughout the book, the authors will show some of their self-talk or logic behind the problem-solving steps. By following along, it is hoped that the student will become proficient in positive, productive self-talk.



This is an example of **negative self-talk!**

1.2 Relationships between Fractions, Decimals, and Percents

One useful concept in all branches of mathematics, as well as many other practical situations, is that one value can be substituted for another provided that the values are **equal** to each other. Determining what values are equal to each other requires understanding how the different values, such as fractions, decimals, and percents, relate to each other.

It should be established that converting an expression from one form to another is not an exercise designed to frustrate the student. Often one way of expressing a value provides the quickest and easiest way to reach the desired result or conclusion.

For example:

> In order to qualify for a special low rate mortgage, the agency, such as the state, may require a minimum down payment of 20% of the selling price of the house. The selling price of the house is \$155,250. One way to find 20% of \$155,250 (there are other methods) is to divide by 5 and get \$31,050, which can be done by inspection. Why? $20\% = \frac{20}{100} = \frac{1}{5}$ and $\frac{1}{5}$ times any value is the same as the value divided by 5.

Understanding how to select the method you understand best is the motivation for learning equivalence of fractions, decimals, and percents. A variety of ways can be used to find solutions to problems. Just as a menu in a restaurant provides us the opportunity to select the meal we want, learning gives us the confidence to develop our own solution to a problem. Just as we address the *same* person by *different* names, i.e. "Mrs. Kugelshopper", "Mom", or "you idiot driver!", we also use *different* names to refer to the *same* quantity. For example, $\frac{1}{2}$, 0.5, and 50% all refer to the *same* amount. We choose different ways of addressing a person depending on the situation. So too, the choice of form (fraction, decimal, or percent) of a particular amount will depend on its usage.

To convert from a fraction to a decimal we merely do the indicated division.

Example: $\frac{1}{4}$ means "1 divided by 4".

$$\text{So,}\quad \frac{1}{4} = 4\overline{)1} = 4\overline{)1.00} = 0.25$$

To convert from a percent to a decimal, first remember that percent means "per hundred" or "divided by 100". So, divide the digits by 100.

Examples: 79% means 79 divided by 100 , or 79 hundredths.

i.e., $79\% = 79 \div 100 = 0.79$

So, $5\% = 0.05$, $27\% = 0.27$,

$100\% = 1.00$, $60\% = 0.60 = 0.6$, and so on.

Remembering that "%" means **divided by 100** will enable you to represent the value correctly.

Examples: $3.5\% = 0.035$ $400\% = 4.00 = 4$ $6.25\% = 0.0625$

To convert from a decimal to a percent, write the decimal as hundredths and substitute the percent sign '%' for the word hundredths.

Examples: 0.67 is 67 hundredths, so $0.67 = 67\%$.

0.3 is 3 tenths or 30 hundredths, so $0.3 = 30\%$.

In general, if you multiply *and* divide a number by the same amount you will not change its value. Hence, if you want to use the percent symbol "%" which indicates *division* by 100 , you must also *multiply* the number by 100 so as not to change the value.

Example: $0.324 = 32.4\%$

A fraction is an indicated division.

Examples: $35 \div 7 = 7\overline{)35} = \dfrac{35}{7} = \tfrac{35}{7}$

$7 \div 35 = 35\overline{)7} = \dfrac{7}{35} = \tfrac{7}{35}$

To convert from a decimal to a fraction, write the decimal digits over the "place", then reduce if necessary.

Examples: 0.379 is read "379 thousandths" which can be written in fraction form as $\tfrac{379}{1000}$.

0.8 is read "8 tenths" which can be written as $\tfrac{8}{10}$ which reduces to $\tfrac{4}{5}$.

1.3 is read "1 and 3 tenths" which can be written as $1\tfrac{3}{10}$.

$0.75 = \tfrac{75}{100} = \tfrac{3}{4}$

$2.24 = 2\tfrac{24}{100} = 2\tfrac{6}{25}$

For those needing a more detailed description of this procedure, consider the following.

Converting a decimal to a fraction:

Step 1: Determine the denominator by writing a 1 in the position of the decimal point and follow with the same number of zeros as the decimal has places.

Step 2: Determine the numerator by writing the whole number that is generated by removing the decimal point.

Example: Convert the value .0125 to a fraction.

Step 1: . 0 1 2 5
↓↓↓↓↓
1 0 0 0 0 — 10,000 is the denominator.

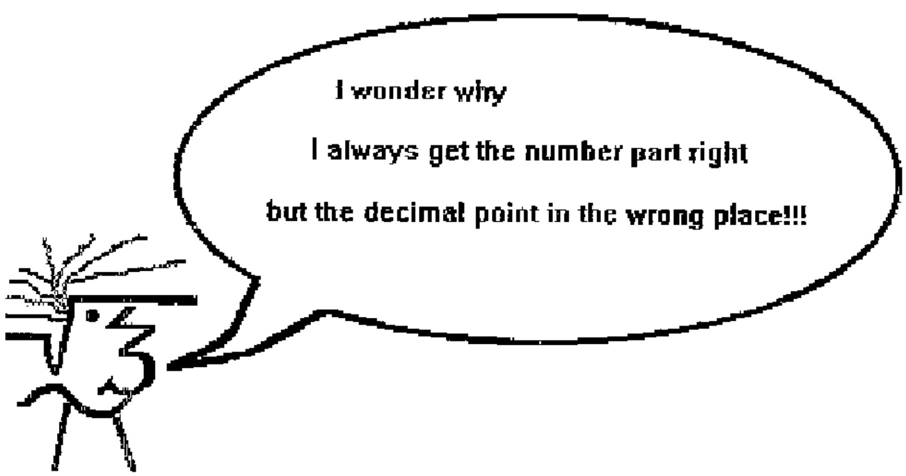
Step 2: 0125 is the same as 125. 125 is the numerator.

$$\text{Hence, } .0125 = \frac{125}{10{,}000} = \frac{1}{80}$$

To convert from a percent to a fraction, write the percent (without the % sign) over 100 and reduce if necessary. You are actually dividing by 100, which is what "percent" means.

Examples: $9\% = \frac{9}{100}$ $20\% = \frac{20}{100} = \frac{1}{5}$

$100\% = \frac{100}{100} = 1$ $300\% = \frac{300}{100} = 3$



If you have the same problem as Irv, you may be relying on rules without understanding. If you understand the relationships between the fraction, decimal, and percent forms, where the decimal point is placed in your answer will become apparent.

Rational Numbers (Equivalences)

Write as fractions. Do NOT solve.

1. $36 \div 9$

2. $8\overline{)40}$

3. $54 \div 6$

4. $9\overline{)63}$

5. $8\overline{)32}$

Divide mentally.

6. $39.76 \div 1000$

7. $45.67 \div 10$

8. $385.8 \div 100$

9. A shipment of 100 hammers costs a hardware store $418. Find the cost of one hammer.

10. A school project will cost a group of 10 students $34. What will be each student's share of the cost?

Write each of the following as a fraction in simplest form AND as a decimal.

11. $14 \div 100$

12. $\frac{35}{1000}$

13. $47 \div 1000$

14. $\frac{68}{100}$

15. $\frac{7}{10}$

16. $31 \div 10000$

17. $\frac{237}{100}$

18. $\frac{145}{1000}$

19. $24 \div 10$

20. $\frac{6}{100}$

Change any fractions to decimals, and change decimals to fractions in simplest form.

21. . 2.87

22. $\frac{7}{20}$

23. 0.462

24. $\frac{5}{8}$

25. 34.0001

Mental Math Challenge 1

Fill in the table below with the missing equivalent fraction (in lowest terms), decimal and/or percent. You will have ten minutes to complete this exercise.

	Fraction	Decimal	Percent
1.	$\frac{1}{2}$		
2.		0.04	
3.			15%
4.		0.1	
5.	$\frac{3}{4}$		
6.		1.8	
7.			70%
8.			340%
9.	$\frac{2}{3}$		
10.			$\frac{1}{2}$%
11.	$\frac{1}{5}$		
12.		0.45	

13.	$\frac{3}{2}$		
14.			40%
15.		0,25	
16.		0.3	
17.		$0.\overline{3}$	
18.			$1\frac{1}{2}\%$
19.	$1\frac{1}{5}$		
20.			60%
21.	$\frac{4}{1}$		
22.			80%
23.			100%
24.	$\frac{7}{25}$		
25.	$\frac{1}{200}$		

Equivalent - having the same value

To determine if rational numbers are *equivalent*, consider the following methods.

To check for equivalence of decimals, write them "the same length". Then, compare.

Examples: Does 1.5 = 1.50 ?

Write 1.5 as a decimal with two decimal places, just as 1.50 is written.

Then, compare the two numbers.

1.50 = 1.50 , so the decimals **are** equivalent.

Does 0.64 = 0.640 ?

Write 0.64 as 0.640 . Then, compare.

0.640 = 0.640 , so the decimals **are** equivalent.

Does 2.31 = 2.301 ?

Write 2.31 as 2.310 . Then, compare.

2.310 $\neq$ 2.301 , so the decimals are **not** equivalent.

Are any of the following pairs of decimals equivalent? (yes/no)

0.69 and 0.689

2.13 and 2.130

0.79 and 0.785

0.236 and 0.2360

12.8 and 13

4.98 and 5

To check for equivalence of fractions, consider either of the following two methods.

Example: Does $\frac{4}{5} = \frac{13}{15}$?

<u>Method 1</u>: For fractions to be equivalent they must reduce to the **same** fraction. Simplify each fraction by reducing to lowest terms (unless they are already reduced), and then inspect them.

Since both of these fractions are given in their lowest terms it is obvious that they are not equivalent.

Hence, $\frac{4}{5} \neq \frac{13}{15}$.

<u>Method 2</u>: Use *cross-products* by multiplying the numerator of each fraction by the denominator of the other fraction. Then, compare their products. If the products are not equal, the fractions are not equivalent.

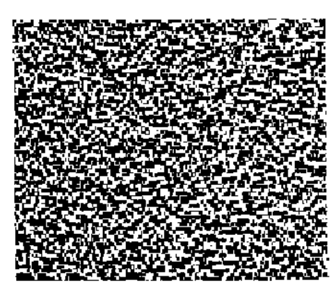
So, to check if $\frac{4}{5} = \frac{13}{15}$, multiply $4 \times 15 = 60$;

then, $5 \times 13 = 65$.

Since $60 \neq 65$, $\frac{4}{5} \neq \frac{13}{15}$.

Example: Does $\frac{3}{6} = \frac{10}{20}$?

If you want to use Method 1 you would have to reduce both fractions. $\frac{3}{6}$ reduces to _____, and $\frac{10}{20}$ reduces to _____. Since they both reduce to the same fraction, then _____.



Are any of the following pairs of fractions equivalent? (yes/no)

$\frac{1}{2}$ and $\frac{6}{12}$

$\frac{2}{3}$ and $\frac{3}{4}$

$\frac{3}{5}$ and $\frac{7}{10}$

$\frac{4}{7}$ and $\frac{20}{35}$

To check for equivalence of **any** pair of rational numbers, the numbers must first be converted to the same form.

Example: Does $\frac{1}{2} = 0.500$?

To determine if these numbers are equivalent, you can either convert the fraction to its decimal form or the decimal to its fraction form; then compare.

[Recall that you divide the numerator by the denominator to convert the fraction to a decimal.]

$\frac{1}{2} =$ _____ $\div$ _____ $= 0.5 = 0.500$

Hence, $\frac{1}{2} = 0.500$.

[Or, recall that you write the decimal digits over the "place" and then reduce to convert the decimal to a fraction.]

$0.500 = \frac{500}{1000} = \frac{5}{10} = \frac{1}{2}$

Hence, $\frac{1}{2} = 0.500$.

Example: Does $\frac{1}{3} = 30\%$?

[Recall that you write the percent over 100 and reduce to convert a percent to a fraction.]

$30\% = \frac{30}{100} = \frac{3}{10}$

Since $\frac{1}{3} \neq \frac{3}{10}$, $\frac{1}{3} \neq 30\%$.

Example: Does $0.5 = \frac{1}{5}$?

$$\frac{1}{5} = 0.2 \text{ and } 0.2 \neq 0.5$$

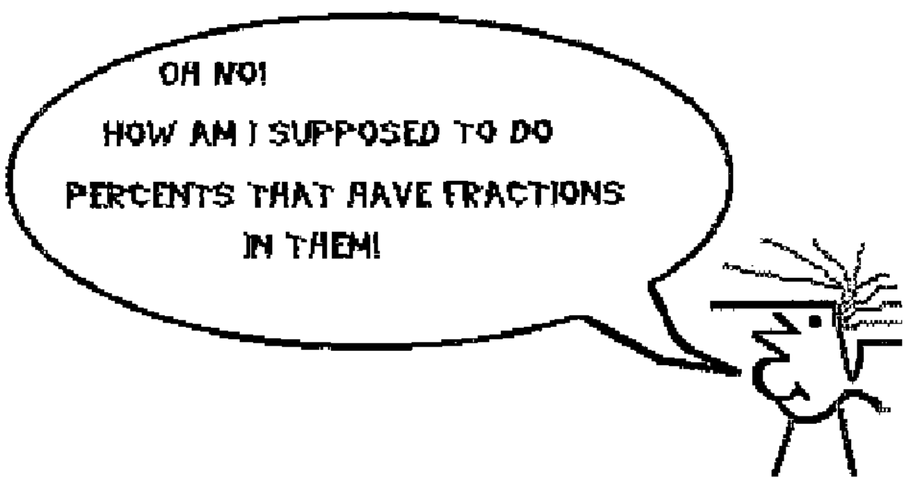
Hence, $0.5 \neq \frac{1}{5}$.

Are any of the following pairs of rational numbers equivalent? (yes/no)

$\frac{1}{2}\%$ and 0.5

0.5% and $\frac{1}{2}\%$

$\frac{1}{2}$ and $\frac{1}{2}\%$



Don't get psyched like our friend Irv. Just remember that the **%** symbol means to **divide by 100!**

Equivalence

For the following pairs of numbers, tell whether each is equivalent, (=), or not equivalent, (≠).

1. 0.25 ________ 25%

2. 0.25 ________ $\frac{1}{5}$

3. $\frac{2}{3}$ ________ 66%

4. $\frac{2}{3}$ ________ 0.6

5. $\frac{2}{3}$ ________ 0.66

6. $\frac{2}{3}$ ________ $0.\overline{6}$

7. 5% ________ $\frac{1}{2}$

8. 5% ________ $\frac{1}{5}$

9. 5% ________ $\frac{1}{20}$

10. 5% ________ 0.05

11. 33% ________ $\frac{1}{3}$

12. 33% ________ 0.33

13. 33% ________ $\frac{33}{100}$

14. 1.8 ________ 180%

15. 1.8 ________ 1.8%

16. 1.8 ________ $1\frac{8}{10}$

17. 1.8 ________ $1\frac{4}{5}$

18. $8\frac{1}{2}$ ________ 8.5

19. $8\frac{1}{2}$% ________ 8.5

20. $8\frac{1}{2}$% ________ 0.085

When did you last compare one value to another to determine which was **greater**? Many times we compare values automatically and, of course, we don't stop to reflect what mathematical concept we are using. Any aware consumer needs to evaluate frequently how to purchase those things that they need or want at the lowest cost. Fortunately, in most cases, the values are expressed in like terms. For example:

Unit price per quart $1.49

Unit price per gallon $5.50

However, since not all comparisons are commonly made, it is necessary to develop those techniques that can be used to reach accurate ways of ordering values.

Symbols can be used to replace the words "is greater than" and "is less than". The symbol resembles the head of an arrow and points to the smaller value. The expression $8 > 5$ is read "eight is greater than five". The expression $2 < 10$ is read "two is less than ten".

You may also need to know how to arrange numbers in order, either from smallest to largest or largest to smallest. This process of arranging is sometimes referred to as *ordering*. Comparing numbers to determine which is larger or smaller is the first step in *ordering*. In order to compare rational numbers, they must first be of the same form (fraction, decimal or percent). Then, you must make sure that they have the **same denominator**.

To compare percents, merely compare the digits themselves since percents are "hundredths" and therefore already have the same denominator.

Example: Is $50\% > 25\%$?

Since $50 > 25$, $50\% > 25\%$.

So, $6\% < 60\%$, $12\% < 12.3\%$, and so on.

Hence, 28% ____ 35%

16.1% ____ 16%

68% ____ 67.9%

To compare decimals, first write them the "same length" (with the same number of decimal places). They will now have the same denominators. Then, compare their digits. [Recall that decimals are decimal fractions and therefore need the same denominators in order to be compared. The **digits** that you compare are actually the **numerators** of the decimals!]

Example: 0.1 ? 0.6

These decimals already have the same number of decimal places, so you only need to compare their digits.

Since $1 < 6$, then $0.1 < 0.6$.

[You are comparing "one tenth" to "six tenths".]

Example: 0.1 ? 0.16

Change the 0.1 to ____. Then, compare the digits.

Since $10 < 16$, then $0.10 < 0.16$.

[You are comparing "ten hundredths" to "sixteen hundredths".]

Hence, $0.1 < 0.16$.

Example: 1 ? 0.3

Change the 1 to _____.

Since $10 > 3$, then $1.0 > 0.3$.

Hence, $1 > 0.3$.

To compare fractions, write them with the same denominators and compare their numerators. You could also write the fractions as decimals and compare. This method may be the best choice for those using calculators.

Example: $\frac{4}{5}$? $\frac{13}{15}$

$\frac{4}{5} = \frac{12}{15}$ [Recall that you can multiply $\frac{4}{5}$ by any fraction that is equivalent to 1 in order to change it to an equivalent fraction with a different denominator. In this case, $\frac{3}{3}$ is used. $\frac{3}{3} \times \frac{4}{5} = \frac{12}{15}$]

Since $\frac{12}{15} < \frac{13}{15}$, then $\frac{4}{5} < \frac{13}{15}$.

Example: $\frac{2}{3}$? $\frac{3}{4}$

Change both to fractions with denominators of 12.

$\frac{2}{3} = \frac{8}{12}$ and $\frac{3}{4} =$ ____

Since $\frac{8}{12} < \frac{9}{12}$, then ____.

To compare any pair of rational numbers, first convert one of them so that they are both in the same form (fraction, decimal, or percent).

Example: 0.25 ? $\frac{3}{8}$

For this problem you could either change 0.25 to a fraction and then compare two fractions, or you could change $\frac{3}{8}$ to a decimal and compare two decimals. Demonstrations of both methods follow.

Method 1:

$0.25 = \frac{1}{4} = \frac{2}{8}$

Since $\frac{2}{8} < \frac{3}{8}$, then $0.25 < \frac{3}{8}$.

Method 2:

$\frac{3}{8} = 0.375$ and $0.25 =$ ____

Since $0.250 < 0.375$, then $0.25 < \frac{3}{8}$.

Example: $\frac{3}{4}$? 0.8

Since $\frac{3}{4}$ is a fraction that can be easily converted to decimal form, go that route.

$\frac{3}{4} = 0.75$ and $0.8 = 0.80$

Since $0.75 < 0.80$, then $\frac{3}{4} < 0.8$.

Example: $\frac{9}{25}$? 0.4

$\frac{9}{25} = 0.36$ and $0.4 =$ ____

Since 0.36 ____ 0.40, then $\frac{9}{25}$ ____ 0.4.

Equivalence & Ordering

Complete each statement by filling in the symbol <, = or >

1. 0.729 0.73

2. $\frac{3}{5}$ $\frac{8}{15}$

3. 35% $\frac{1}{4}$

4. 3 2.9

5. $\frac{3}{4}$ $\frac{15}{20}$

6. 0.2 21%

7. 0.03 0.030

8. $\frac{2}{3}$ $\frac{5}{8}$

9. 100% 3.1

10. 0.41 0.4

11. $\frac{11}{12}$ $\frac{5}{6}$

12. 0.5 $\frac{2}{3}$

13. 1.08 1.80

14. 22% $\frac{1}{5}$

15. 2 2.0

Equivalence and Ordering the Sequel

a) 0.005	e) 50	i) $\frac{1}{2}$
b) 0.05	f) 500	j) $\frac{1}{5}$
c) 0.5	g) $\frac{1}{20}$	k) $\frac{1}{200}$
d) 5	h) $\frac{1}{50}$	l) $\frac{1}{500}$

Choose the letter(s) for the answer(s) that will make the following statements true.

1. 5% = ____________
2. 5% > ____________
3. 5% < ____________
4. 50% = ____________
5. 50% > ____________
6. 50% < ____________
7. 500% = ____________
8. 500% > ____________
9. 500% < ____________
10. 0.5% = ____________
11. 0.5% > ____________
12. 0.5% < ____________

Ordering

Choose the ***largest*** from each of the following pairs of rational numbers.

1. 0.25 or $\frac{1}{5}$

2. $\frac{2}{3}$ or 66%

3. $\frac{2}{3}$ or 0.6

4. $\frac{2}{3}$ or 0.66

5. 5% or $\frac{1}{2}$

6. 5% or $\frac{1}{5}$

7. 33% or $\frac{1}{3}$

8. 1.8 or 1.8%

9. $8\frac{1}{2}$% or 8.5

1.3 Ratio and Proportion

Ratio - a number written as the quotient of two integers, i.e. written in the form of a fraction. It compares two numbers by division.

Example: The ratio $\frac{2}{3}$ would mean "two compared to three" or merely "2 to 3".

Example: The ratio of 6 to 11 would be expressed as $\frac{6}{11}$.

A *ratio* may be written using the symbol " : ".

Example: The ratio of 5 to 8 would be expressed as 5 : 8 .

A *ratio* should always be written in its simplest form; that is, as a reduced fraction.

Example: The ratio of 4 to 10 would be $\frac{4}{10}$, or $\frac{2}{5}$ in simplest form.

Ratios can compare **like** quantities.

Example: The ratio of 9 inches to 14 inches would be $\frac{9}{14}$. The unit "inches" need not be written.

Example: The ratio of 20 lb. to 50 lb. would be $\frac{20}{50}$ or $\frac{2}{5}$.

Ratios can compare **unlike** quantities.

If the quantities to be compared are *unlike*, it may be possible to convert one of the units and express them as *like* quantities.

Example: To express the ratio of 2 feet to 7 inches, first convert the number of feet into a number of inches.

Since there are 12 inches in 1 foot, 2 feet would equal 24 inches.

Hence, $\frac{2 \text{ feet}}{7 \text{ inches}} = \frac{24 \text{ inches}}{7 \text{ inches}} = \frac{24}{7}$.

Example: To express the ratio of 10 hours to 2 days, first convert the number of days into a number of hours.

2 days = 48 hours

Hence, $\frac{10 \text{ hours}}{2 \text{ days}} = \frac{10 \text{ hours}}{48 \text{ hours}} = \frac{10}{48} = \frac{5}{24}$.

There may be times when the *unlike* quantities to be compared cannot have their units converted to make them *like* quantities.

Example: To express the ratio of 20 ounces to 50 cents, no conversion can be made. However, it must still be written in simplest form. Also, since the units are different they must be written in the ratio.

Hence, $\frac{20 \text{ ounces}}{50 \text{ cents}} = \frac{2 \text{ ounces}}{5 \text{ cents}}$.

Example: A 6 lb. roast to serve 24 persons would be expressed as

$\frac{6 \text{ lb.}}{24 \text{ persons}} = \frac{1 \text{ lb.}}{4 \text{ persons}}$.

Write as ratios in simplest form.

16 to 12

100 to 90

20 yd. to 24 yd.

8 in. to 3 ft.

9 dimes to 3 quarters

A soccer team wins 7 of its 18 games played.

Write the ratio of games won to games played.

Write the ratio of games won to games lost.

Proportion - a statement that two ratios are equal.

Example: $\frac{3}{4} = \frac{6}{8}$ is read "three is to four as six is to eight".

Each number in a *proportion* is called a **term** of the *proportion*.

Use *cross-products* to check if a *proportion* is **true**.

Example: For $\frac{3}{4} = \frac{6}{8}$, $3 \times 8 = 24$ and $4 \times 6 = 24$.

Hence, since the cross products are equal, $\frac{3}{4} = \frac{6}{8}$ is a *true proportion*.

[This is the same procedure you used to check for equivalence of fractions.]

Why is it true that, in a true proportion, the cross products must be equal?

The form of a proportion is that of two fractions set equal to each other. Any fraction can be written differently without changing its value provided the numerator and denominator are multiplied by the same value. For example:

$\frac{2}{5} \times \frac{10}{10} = \frac{20}{50}$

Since $\frac{10}{10}$ is a form of 1, the **value** of $\frac{2}{5}$ stays the same when expressed as the fraction $\frac{20}{50}$.

So, if $\frac{2}{5} = \frac{4}{10}$, then $\frac{2 \times 10}{5 \times 10} = \frac{4 \times 5}{10 \times 5}$.

The denominators of the fractions become equal because they are multiplied by each other. The numerators then are multiplied by the value that was multiplied by its denominator, or the other fraction's denominator, creating the perception of cross-multiplication or *cross-products*. It should be understood that the process called *cross-products* is applied when a proportion is being used to solve a problem. Understanding proportions provides another tool for solving problems.

When could the concept of proportions be used?

Using proportions with a road map can enable you to approximate the number of miles traveled and the travel time. Using proportions with a recipe can enable you to adjust the ingredients to match the serving size you'll need. Section 1.4 will show even more uses for proportions.

To find a missing *term* of a proportion consider the following two methods.

Method 1: This *cross-products method* can be used for **any** proportion.

Example: $\frac{3}{9} = \frac{?}{6}$

To find the missing term, first replace the "?" with an "x": $\frac{3}{9} = \frac{x}{6}$.

Next, cross-multiply and set the two products equal to each other:

$9 \times x = 3 \times 6$.

However, it is not necessary to write the multiplication sign between the 9 and the x. It is simply written as $9x = 3 \times 6$. So, $9x = 18$.

To find the value for x, divide the 18 by the 9.

[You are asking yourself "9 multiplied by what number equals 18 ?"]

Hence, $x = 2$, which is the missing term in the original proportion.

This answer can be **checked** by replacing the x with the 2 in the original proportion and then using *cross-products* to see if it is a **true proportion**. Since both cross-products equal 18, it *is* a true proportion and the answer 2 is correct.

Example: $\frac{6}{x} = \frac{9}{12}$

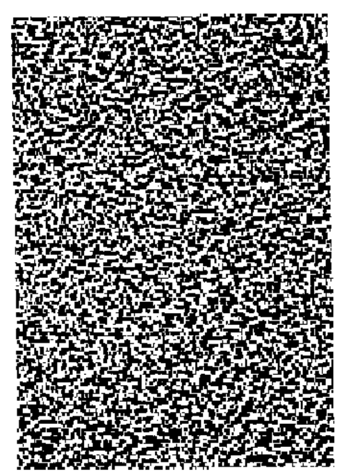
$9x =$ _____

Hence, $x =$ ____ .

Example: $\frac{x}{12} = \frac{0.5}{2}$

____ $x =$ ____

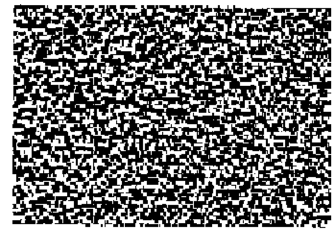
Hence, $x =$ ____.



Find the missing term of each proportion.

$$\frac{4}{7} = \frac{12}{x}$$

$$\frac{10}{x} = \frac{0.2}{4}$$



Method 2: A "short-cut" method that we refer to as the *sideways method* can be used whenever one of the terms in one ratio is a factor of the term along side of it.

Example: $\frac{7}{8} = \frac{x}{32}$

Note that 8 is a factor of 32.

Since $8 \times 4 = 32$, multiplying 7×4 will give you your missing term.

Hence, $x = 28$.

[What you are really doing its multiplying the ratio $\frac{7}{8}$ by $\frac{4}{4}$, a form of 1, to yield an equivalent ratio $\frac{28}{32}$.]

Example: $\frac{8}{9} = \frac{24}{x}$

Note that 8 is a factor of 24.

Since $8 \times 3 = 24$, $9 \times 3 = 27$.

Hence, $x = 27$.

Example: $\frac{12}{x} = \frac{6}{7}$

$6 \times 2 = 12$ and $7 \times 2 = 14$.

Hence, $x = 14$.

Example: $\frac{x}{18} = \frac{20}{24}$

Note that 18 is **not** a factor of 24 . But, the ratio

$\frac{20}{24}$ can be written in simplest form as the ratio $\frac{5}{6}$.

You now have an equivalent proportion $\frac{x}{18} = \frac{5}{6}$,

in which 6 is a factor of 18 ! So, you can use the *sideways method!* Since $6 \times 3 = 18$, multiply $5 \times$____. Hence, $x =$____.

Example: $\frac{x}{3} = \frac{7}{18}$

Note that 3 is a factor of 18 , but **be careful!**

Since the missing term is located in the ratio that has the **smaller number** , 3 in this case, you will **not** be **multiplying** the ratio $\frac{7}{18}$ by **anything!** That would only give you an equivalent ratio that would have **larger** terms in it! Instead you will be **dividing by a form of 1** , $\frac{6}{6}$ in this case, to yield an equivalent ratio with **smaller** terms.

Since $18 \div 6 = 3$, you must also divide 7 by 6 .

This result, $1\frac{1}{6}$ or $1.1\overline{66}$, is the missing term x .

Hence, the completed proportion is $\frac{1\frac{1}{6}}{3} = \frac{7}{18}$.

It may look strange, but it checks. Just cross-multiply and see! Both cross-products equal 21 .

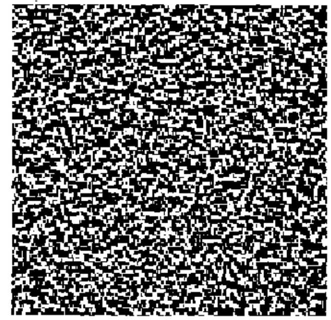
[By the way, you will get the same result if you use Method 1.]

Find the missing term of each proportion using the *sideways method*.

$\frac{6}{4} = \frac{x}{2}$

$\frac{x}{36} = \frac{13}{3}$

$\frac{2}{9} = \frac{x}{36}$



1.4 Application Problems

Proportions can be used to solve word problems.

Example: If the ratio of girls to boys in a science club is 4 to 5, find the number of boys in the club if there are 16 girls.

[The ratio stated means that there are 4 girls for every 5 boys.]

To solve this problem, set up a proportion.

$$\frac{4 \text{ girls}}{5 \text{ boys}} = \frac{16 \text{ girls}}{x \text{ boys}} \text{ or simply } \frac{4}{5} = \frac{16}{x}.$$

The solution to the proportion, 20, is the solution to the problem. Hence, there must be 20 boys in the science club.

Example: Andrea received, in a letter, a picture of her brother Frank and his three children. She has not seen her brother and his family in a while. She noticed that the children have really grown since she saw them last. Her brother is 6 feet tall. In the picture, his image measures ≈ (approximately) 4 inches. His son Louis' image measures ≈ 2 inches. His daughter Tracy's image measures ≈ 3 inches, and his son Mark's image measures ≈ $3\frac{1}{2}$ inches. Approximately how tall are the children?

Step 1: Since her brother's height is known, the ratio of $\frac{6 \text{ feet}}{4 \text{ inches}}$

can be used to find the height of each of the children.

Step 2: If $\frac{6'}{4''} = \frac{\text{Louis' height}}{2''}$, how tall is Louis?

$\frac{6'}{4''} = \frac{x'}{2''}$, $4x = 12$, so $x = 3$.

Hence, Louis must be 3 feet tall.

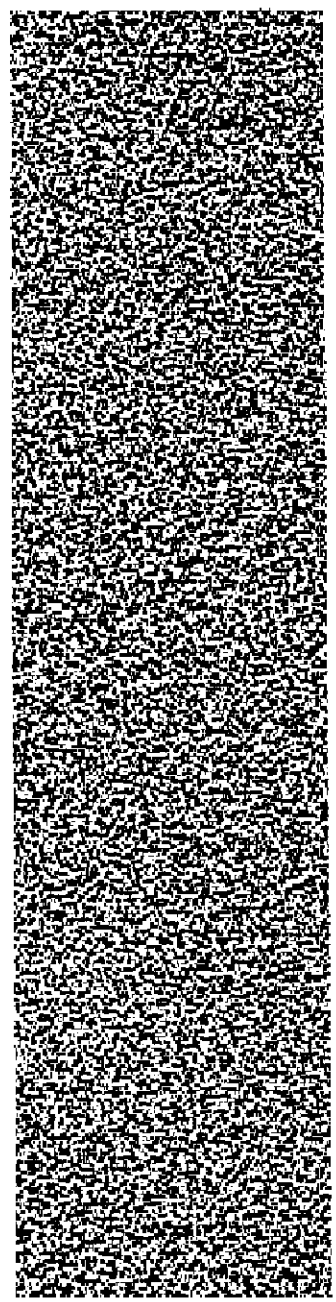
To find Tracy's and Mark's heights we can use

$\frac{6'}{4''} = \frac{\text{Tracy's height}}{\_\_\_\_ \text{inches}}$ and $\frac{6'}{4''} = \frac{\text{Mark's height}}{\_\_\_\_ \text{inches}}$

Hence, $\frac{6'}{4''} = \frac{x'}{3''}$ gives the solution _____ for

for Tracy's height, and $\frac{6'}{4''} = \frac{x'}{3\frac{1}{2}''}$ gives the

solution _____ for Mark's height.



Once we had found Louis' height, could we have used the ratio of *his* height to the measure of *his* picture image to find Mark's height? _____

In Andrea's picture, all four of the people were *standing* in front of their home. Why is this fact important to her information? We measure a person's height when he/she is _____.

Suppose we are on vacation and we want to capture for our friends back home not only the beauty of a sculpture, but the size (which is humongous). We could make certain that someone or something, the size of which we know, is also in the picture as a reference. We are using the properties of _____ to enhance our memories.

Proportions

1. My friend is 6 ft tall and casts a shadow 8 ft long. If a nearby flagpole casts a shadow 12 ft long, at the same time of day, how tall is it?

2. If oranges are on sale 10 for $2.00, how much would 25 cost?

3. A basketball player scores 10 goals in the first 4 games of the season. How many goals would you expect him to get in a 10 game season?

4. If $\frac{1}{2}$ inch is used to represent 100 miles on a map, how far would a trip be that is 3 inches on the map?

5. If 8 quarts of paint covers 900 ft^2, how much would you need to cover 225 ft^2?

6. A store has soup on sale 2 cans for 99 cents, how much would a case of 24 cans cost?

7. If an 8 lb roast serves 18 people, what size is needed to serve 27 people?

8. If Tom can clean 7 rooms in 8 hrs, how many could he clean in 32 hrs?

9. If Mary can read 5 books in 35 days, how many would she read in 28 days?

10. If John can make a 20 million dollar profit in 24 months, how much can he make in 18 months?

Let's consider some **choices** you are now prepared to make.

Example: Which would you rather have, a $10\frac{1}{2}\%$ raise or $\frac{1}{5}$ raise?

Before you do any math at all, it may be wise to ask yourself whether you would like a *larger or smaller* raise. Of course, one would choose the ________.
Now we have effectively restated the problem as a problem in *ordering*. The problem can now be restated in mathematical terms.

Which *is larger* $10\frac{1}{2}\%$ or $\frac{1}{5}$?

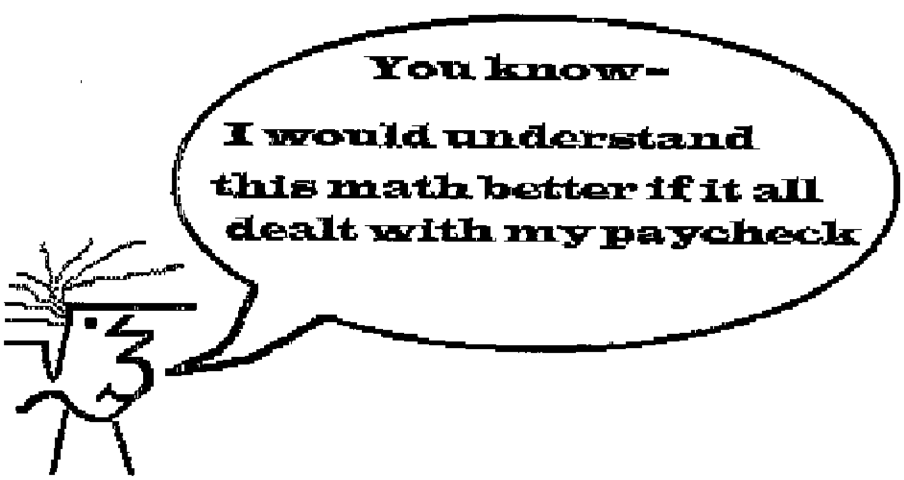
There are several different ways one could compare these quantities. Whatever method you chose must end with comparing units that are the same.

One method is to convert $\frac{1}{5}$ to the percent form .

$\frac{1}{5} = \frac{?}{100}$

So $\frac{1}{5} =$ _______%

Since 20% is larger than $10\frac{1}{2}\%$, the best choice for the raise is _________.



Example: Which would you rather pay, a 5% sales tax or a $\frac{1}{5}$ sales tax?

Here again, the problem must be stated mathematically. Ask yourself, as in the first example, whether you would want a larger or smaller tax.

Given the choice, a ________ tax would be best.

This problem can now be restated.

Which *is smaller*, 5% or $\frac{1}{5}$?

By comparing these quantities, ________ is the smaller. Therefore, going back to the original problem, one would choose to pay the ________ sales tax.

[Notice, if you did not read the problem accurately to determine what you were actually being asked to find you could do all the computations accurately and still choose the *wrong* answer.]

Example: Which bonus would you rather have? 7% or $\frac{1}{7}$

First, would you rather have the larger or smaller amount? __________

Restate the problem.

Which is __________ 7% or $\frac{1}{7}$?

The preferred bonus is ________

Choices

1. Which interest rate would you rather receive?

   $6\frac{1}{2}\%$ $\frac{1}{8}$

2. Which sales tax rate would rather pay?

   8% $7\frac{1}{2}\%$

3. Which portion of your rich uncle's estate would you rather inherit?

   $\frac{2}{3}$ 60%

4. Which amount would you rather get off a purchase?

   $\frac{1}{2}$ 40%

5. Which pay raise would be the best choice?

   $\frac{2}{3}$ $\frac{4}{7}$

The ***sideways method*** that was demonstrated to you for finding a missing term of a proportion can also be applied to conversions of certain fractions. This can work well when converting fractions to other equivalent fractions, decimals, or percents. The key here is to focus on the denominator of the original fraction.

Any fraction can be converted to an equivalent fraction containing a larger numerator and denominator if the denominator of the original fraction is a factor of the denominator of the new fraction.

Example: To convert $\frac{3}{4}$ to an equivalent fraction with a denominator of '8', determine what you would have to multiply 4 by to get 8. Then, multiply the 3 by the same number to get the numerator of the equivalent fraction. This "key number", 2 in this case, can easily be used to get the answer **mentally**.

$\frac{3}{4} = \frac{?}{8}$ becomes $\frac{3}{4} = \frac{6}{8}$ with hardly any work.

Notice that this format of setting equivalent fractions equal to each other is none other than a proportion!

$\frac{3}{4} = \frac{?}{8}$ is really the same as $\frac{3}{4} = \frac{x}{8}$

Any fraction whose denominator is a factor of a *power of 10* can be converted to a decimal by using the *sideways method*. [Recall that powers of 10 are 10, 100, 1000, 10000, and so on.] This works because decimal places are *powers of 10*!

Example: To convert $\frac{4}{5}$ to a decimal, first note that the denominator '5' is a factor of 10 (which is a *power of 10*). Now you can apply the *sideways method* as you **first** convert $\frac{4}{5}$ to an equivalent **fraction** whose denominator is 10.

$$\frac{4}{5} = \frac{?}{10}$$

Using 2 as your 'key number', the missing numerator must be 8.

Thus, $\frac{4}{5} = \frac{8}{10}$, and since $\frac{8}{10}$ is equivalent to 0.8, $\frac{4}{5} = 0.8$.

Example: Convert $\frac{7}{20}$ to a decimal.

$$\frac{7}{20} = \frac{?}{100}$$

$$\frac{7}{20} = \frac{35}{100} \text{ and since } \frac{35}{100} = 0.35,\ \frac{7}{20} = 0.35.$$

Any fraction whose denominator is a factor of 100 can be converted to a percent by using the *sideways method*.

Example: To convert $\frac{9}{25}$ to a percent, first note that the denominator '25' is a factor of 100 .

You can once again apply the *sideways method* as you **first** convert $\frac{9}{25}$ to an equivalent **fraction** whose denominator is 100 .

$$\frac{9}{25} = \frac{?}{100}$$

$$\frac{9}{25} = \frac{36}{100}, \text{ and since } \frac{36}{100} = 36\%,\ \frac{9}{25} = 36\%.$$

Example: Convert $\frac{11}{20}$ to a percent.

$$\frac{11}{20} = \frac{?}{100}$$

$$\frac{11}{20} = \frac{55}{100}, \text{ and since } \frac{55}{100} = 55\%,\ \frac{11}{20} = 55\%.$$

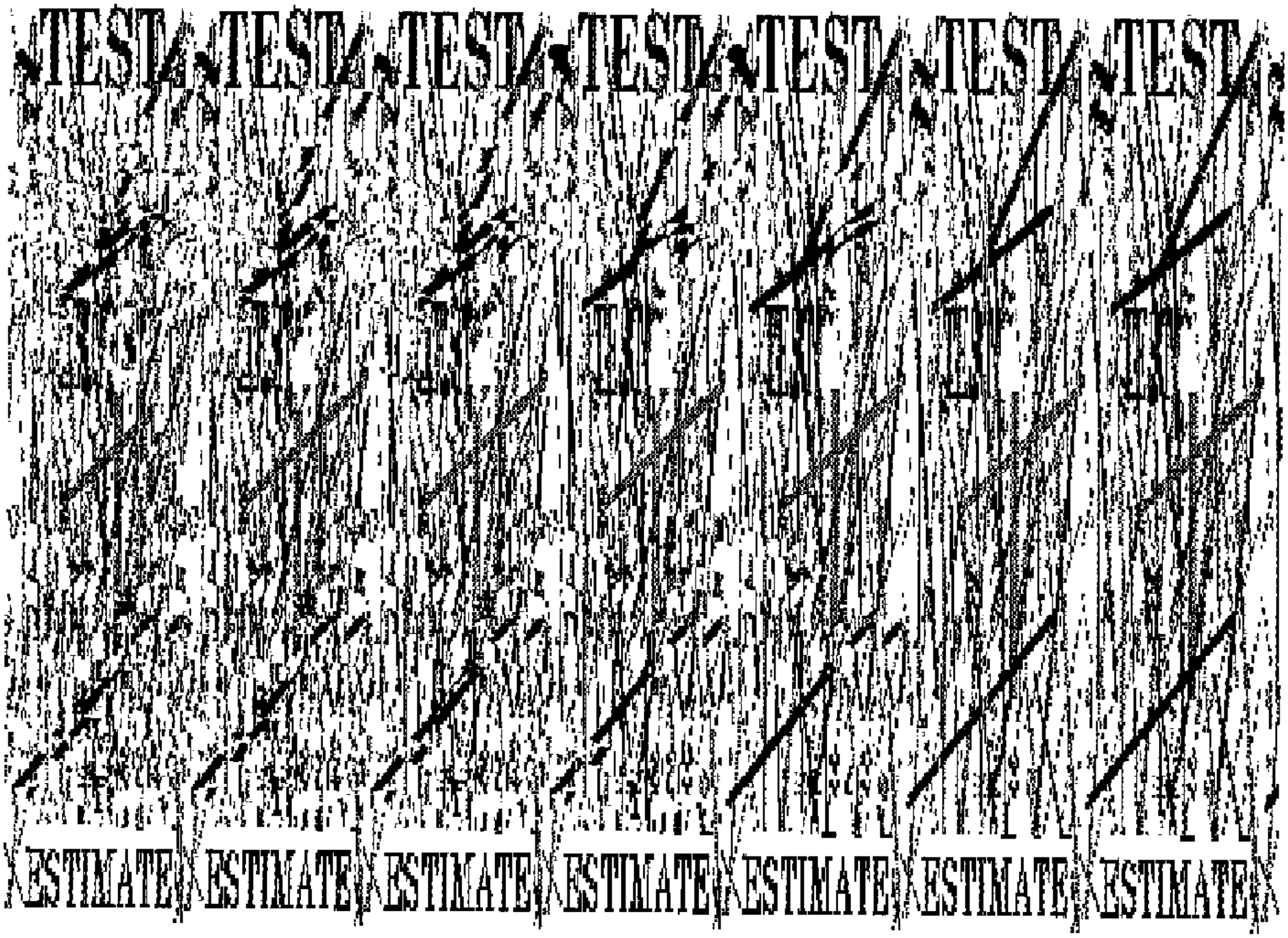
Remember that the *sideways method* is a **short-cut** and each of the examples above can be solved by other methods that have been demonstrated in this unit.

Convert each of the following.

$\frac{2}{9}$ to a fraction with a denominator of '36'

$\frac{17}{50}$ to a percent

$\frac{8}{500}$ to a decimal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Unit 2

2.1 Introduction

Too often the techniques of approximation and estimation are not considered important in applying mathematical concepts. Certainly an exact, accurate answer is most times the focus, but often careless mistakes could be prevented if the approximation or range of the result were predetermined. The techniques for making intelligent approximations can be especially useful to improve test performance. How often is an answer incorrect because the decimal point is in the wrong place, or not included when it is necessary? Understanding how to recognize a realistic answer helps to avoid an extremely incorrect selection. Sometimes an exact answer is not necessary and even inconvenient to compute. Again, the techniques of approximation and estimation are invaluable. It is important to realize that approximation is not random guessing.

Approximate value - a value that is close to the exact value

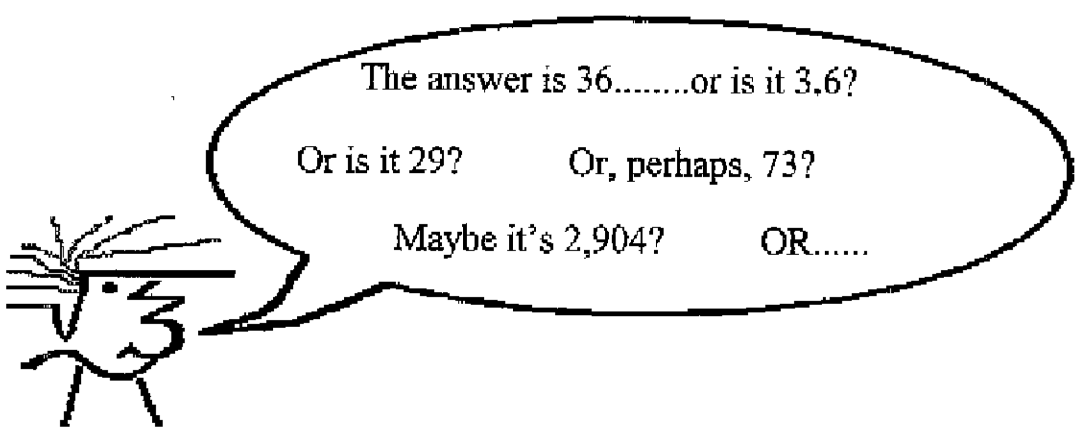
The symbol for "approximates" or "is approximately" is '$\approx$'. It resembles an equals sign, but it shows that the value is not exactly equal.

Example: $\frac{15.9}{8} \approx 2$

Estimate - the method used to judge or arrive at an *approximate value*

Example: $\frac{15.9}{8}$

15.9 is almost equal to 16, and since $16 \div 8 = 2$, $15.9 \div 8 \approx 2$.



Irv is just guessing. Approximations and estimations are not guesses.

2.2 Approximations of Whole Numbers

The concept of **rounding** is used when a value needs to be expressed in specific terms. Although many different rounding methods can be used, it is common to round to the nearest of the specific value designated.

Example: *Round* 167 to the nearest ten.

To what number of tens is the number 167 closest?

Step 1: What number of tens are possible choices?

167 is between 160 (16 tens) and 170 (17 tens).

Step 2: Find the difference of the value and each choice.

167 is 7 units away from 160 but only 3 units away from 170 .

Hence, 167 is closest to 170 or 17 tens.

So, 167 rounded to the nearest ten is 170 .

When *rounding* is used, the approximate value that best represents the given value is that one that is closest. However, this process would lead to doubt when a value is the **same** distance from the smaller as from the larger value.

Example: Round 3,275 to the nearest ten.

3275 is between 3270 (327 tens) and 3280 (328 tens).

3275 is 5 units away from both 3270 **and** 3280 .

In this case, it is common procedure to choose the larger value.

Hence, 3,275 rounded to the nearest ten is 3,280 .

When *rounding* is completed, all values following the position to be rounded become zeros.

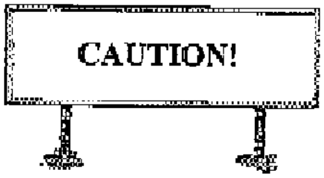
A short-cut for rounding follows.

The first step is to look at the value immediately to the right of the position to be rounded. If this value is equal to or greater than 5 , round 'up'. If it is less than 5 , round 'down'.

Example: Round 8,469 to the nearest thousand.

The value in the thousands place is 8 , so you inspect the value to the right of the 8 which is the 4. Since 4 is less than 5 , round 'down'.

Hence, 8,469 $\approx$ 8,000 .



When rounding, the value immediately to the right of the position to be rounded is the **only** value that affects the result.

Example: Round 6,749 to the nearest hundred.

6749 is between 6700 and 6800 .

6749 is 49 units away from 6700 but 51 units from 6800 .

Since it is closer to 6700 , 6,749 $\approx$ 6,700 .

If you try the ***short-cut*** for this problem, you would have to be careful not to 'look too far to the right'. In other words, the 9 does **not** 'round up' the 4 to a 5 which in turn 'rounds up' the 7 to an 8 . This would result in the **incorrect** answer of 6,800 !

One method of applying this short-cut that would help to prevent an error like the one shown above would be to use the notations that follow.

Example: Round 75 to the nearest ten.

First underline the value in the tens place (the position to be rounded).

$\underline{7}5$

Then, put a check mark '$\checkmark$' over the value immediately to the right of it.

$\underline{7}\overset{\checkmark}{5}$ Hence, 75 $\approx$ 80 .

Example: Round 953 to the nearest ten.

$9\underline{5}\overset{\checkmark}{3}$ Hence, 953 $\approx$ 950 .

Example: Round 4,452 to the nearest thousand.

$\underline{4}\overset{\checkmark}{4}52$ So, the **underlined** '4' is the only value you would possibly round and the **checked** '4' is the only value you would inspect.

Hence, 4,452 $\approx$ 4000 .

Example: Round 887,000 to the nearest ten thousand.

$887{,}000 \approx 890{,}000$.

Round each of the following to the indicated place.

2,876 to the nearest ten

11,249 to the nearest hundred

6,500 to the nearest thousand

Rounding can be used to *estimate sums*.

Example: The sum of $287 + 413 + 875$ can be estimated by first rounding each of the numbers to the **largest place value.** In this case, you would round to the nearest hundred.

Thus 300 , ____ , and ____ will be the approximations used to estimate.

$300 + 400 + 900 = 1600$, the estimated sum.

Hence, $287 + 413 + 875 \approx 1600$.

If you'd like, you could compute the exact sum and then compare it with the estimated sum. In this case, you would compare 1575 with 1600 . This should indicate that the estimate seems reasonable.

This procedure can be particularly useful when shopping!

Estimate the following sums.

$1{,}567 + 439$

$23 + 276 + 721$

\$399 + \$599 + \$1299

$23{,}900 + 157 + 49 + 8$

Rounding can also be used to *estimate products.*

Example: The product of 412×289 can be estimated by first rounding the numbers to the **largest place value**. In this example, round to the nearest hundred.

Thus, ______ and ______ will be the approximations used to estimate.

$400 \times 300 = 120{,}000$, the estimated product.

Hence, $412 \times 289 \approx 120{,}000$.

If in the preceding example you thought to yourself that the only way you could have multiplied the 400 by the 300 would be by doing **a lot of work** involving **a lot of zeros**, think again. **There is a better way!** An explanation of this shorter procedure follows. It will help you to multiply by both *powers of ten* and *multiples of ten.*

When any value is multiplied by zero, the result is zero. When any value is multiplied by one, the result is the original value.

Example: $6 \times 0 = 0$ and $8 \times 1 = 8$

All powers of 10 are expressed as 1 followed by a number of zeros equal to the value of the exponent.

Example: $10^3 = 1000$

When a value is multiplied by a power of 10, the multiplication can be done by inspection since the result can be easily anticipated.

Example:

```
   2875
 × 100
   0000
  0000
 2875
 287500
```

Note that there are **two** zeros in the power of 10 that is the multiplier. Those zeros will simply become the digits in the last **two** places following the original number since that part of the result reflects the original value.

Examples: 89 × 1000 = 89,000

686 × 10,000 = 6,860,000

One way that these problems are sometimes written follows.

Examples:

$$\begin{array}{r} 89 \\ \times \underline{1000} \\ 89{,}000 \end{array} \quad \text{and} \quad \begin{array}{r} 686 \\ \times \underline{10000} \\ 6{,}860{,}000 \end{array}$$

This leads into a short-cut for multiplying by a **multiple** of 10.

Example: To multiply 213 × 400 , set it up vertically as in the previous example.

$$\begin{array}{r} 213 \\ \times \underline{\ \ 400} \\ 85{,}200 \end{array}$$

What you are really doing is multiplying the 213 by the 4 . Then, the **two** zeros in the 400 , the *multiple* of 10 , simply become the digits in the last **two** places in the answer following 852 (the product of 213 and 4).

Estimate the following products.

26 × 1,000

319 × 10,000

439 × 6874

Rounding And Estimating

Round each of the following to the indicated place.

1. 218 nearest ten

2. 5673 nearest thousand

3. 5.7642 nearest thousandth

4. 34.592 nearest hundredth

5. 345 nearest hundred

6. 0.67 nearest tenth

7. 9,372 nearest hundred

8. 8,956 nearest hundred

9. 3.398 nearest tenth

10. 23,455 nearest ten

11. 932 nearest thousand

12. 0.3982 nearest tenth

Round to two decimal places.

13. 398.486

14. 9,388.2145

15. 0.237

Estimate each of the following **sums** by first rounding to the largest place value. Then do the addition and compare to your estimate.

16.
475
294
206

17.
2429
312
1034

18.
4962
1297
2930

Estimate each of the following **products** by first rounding to the largest place value. Then do the multiplication and compare to your estimate.

19.
84
× 57

20.
989
× 234

Mental Math Challenge 2

Fill in the table below with the missing equivalent fraction (in lowest terms), decimal and/or percent. You will have ten minutes to complete this exercise.

	Fraction	Decimal	Percent
1.			0.5%
2.			28%
3.	$\frac{1}{1}$		
4.		0.8	
5.		4.0	
6.	$\frac{3}{5}$		
7.		1.2	
8.		0.015	
9.			$33\frac{1}{3}\%$
10.	$\frac{3}{10}$		
11.			25%
12.		0.4	

13.		1.5	
14.			45%
15.			20%
16.		0.005	
17.			$66\frac{2}{3}\%$
18.	$3\frac{2}{5}$		
19.		0.7	
20.	$1\frac{4}{5}$		
21.		0.75	
22.			10%
23.	$\frac{3}{20}$		
24.	$\frac{1}{25}$		
25.		0.5	

Estimations

Estimate the answers to each of the following problems by rounding each number before the operation is performed. **This exercise will be timed. Do not begin until you are instructed. Work as quickly as you can.**

1. $508 + 678 \approx$ ____________________

2. $12{,}567 \div 18 \approx$ ____________________

3. $82{,}499 - 67{,}745 \approx$ ____________________

4. $569 \times 1986 \approx$ ____________________

5. $753 + 987 + 445 \approx$ ____________________

6. $22{,}649 - 2{,}834 \approx$ ____________________

7. $360{,}823 \div 20{,}871 \approx$ ____________________

8. $249{,}342 \times 251{,}008 \approx$ ____________________

9. $\$8{,}099 + \$7{,}909 + \$1{,}903 \approx$ ____________________

10. $92{,}344{,}005 \times 782 \approx$ ____________________

2.3 Approximations of Fractions and Decimals

Decimals can also be rounded using the procedure demonstrated in Section 2.2.

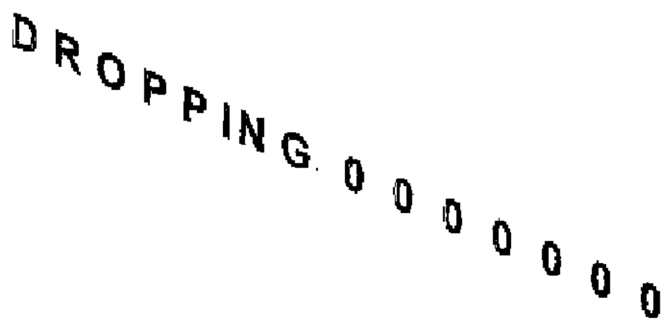
Example: Round 0.157 to the nearest tenth.

> 0.157 is between 0.100 and 0.200 .
>
> 0.157 is 57 units from 0.100 and 43 units from 0.200 .
>
> Since 0.157 is closest to 0.200 , 0.157 $\approx$ 0.200 .

You can also use the short-cut for this problem. The value in the tenths place is the 1 , so you inspect the value to the right of the 1. Since this value is a 5 , round 'up'.

Example: Round 89.3728 to the nearest thousandth.

> The value in the thousandths place is a 2 . Inspection of the 8 to the right of it indicates that you should round the 2 up to a 3 . Hence, 89.3728 $\approx$ 89.3730 .



The answer will not be altered by omitting or dropping the zeros to the extreme right. These zeros are sometimes referred to as **insignificant digits**. Hence, in the preceding examples the answers could have been written as: 0.157 $\approx$ 0.2 and 89.3728 $\approx$ 89.373 . This supports a common practice in mathematics of simplifying expressions to reduce the number of required symbols.

There will be times when the directions in a problem will state that you are to round to 'a certain number of decimal places'. This type of wording makes the problem even easier to do since you don't have to determine what position is being referred to .

Example: Round 8.731 to two decimal places.

> This means that the answer should have only two decimal digits in it.
>
> After inspecting the value 1 , you should conclude that 8.731 $\approx$ 8.73 .

Round each of the following.

12.428 to the nearest tenth

601.386 to the nearest hundredth

0.1439 to three decimal places

0.0495 to the nearest tenth

Rounding can be used to *estimate sums, differences, products, and quotients* of decimals.

Example: The sum of $3.07 + 4.81 + 5.16$ can be found by first rounding each of the decimals to the nearest whole number. So, 3 , _____, and _____ would be the approximations. Hence, $3.07 + 4.81 + 5.16 \approx$ _____.

Example: The product of 4.87×11.3 could be estimated by finding the product of _____ $\times$ 11 . Hence, $4.87 \times 11.3 \approx$ _____.

Example: The quotient of $12.08 \div 3.79$ could be estimated by finding the quotient of _____ $\div$ _____. Hence, $12.08 \div 3.79 \approx$ _____.

Estimate each of the following.

$59.487 + 0.85$

5.28×19.9432

\$ 24.98 + \$15.49 + \$.99

Sums, differences, products, and quotients of fractions and mixed numbers can also be *estimated*.

Example: The sum $4\frac{5}{8} + 3\frac{11}{13}$ could be estimated by finding the sum of $5 + 4$ which are approximations of the original numbers. Hence, $4\frac{5}{8} + 3\frac{11}{13} \approx 9$.

Example: The difference of $5\frac{12}{17} - 3\frac{3}{8}$ could be estimated by finding the difference of $16 - 3$, their approximations. Hence, $5\frac{12}{17} - 3\frac{3}{8} \approx 13$.

Example: The product of $2\frac{4}{7} \times 6\frac{1}{9}$ could be estimated by finding the product of _____ and _____.
Hence, $2\frac{4}{7} \times 6\frac{1}{9} \approx$ _____.

Estimate each of the following.

$9\frac{4}{5} + 2\frac{1}{3}$

$9\frac{4}{5} - 2\frac{1}{3}$

$9\frac{4}{5} \times 2\frac{1}{3}$

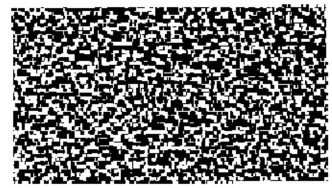
$9\frac{4}{5} \div 2\frac{1}{3}$

Estimation can be used in application problems.

Example: On a weekend getaway Holly's expenses were \$132.48 for lodging, \$87.89 for food, \$15.75 for gas and tolls, and \$47.40 for miscellaneous expenses. Estimate Holly's total expenses.

Rounding each amount to the nearest dollar would give the following approximations:
\$132 + \$88 + _____ + _____ which would give an estimate of_____.

For a more "ball park figure" you could round each amount to the nearest ten dollars which would give approximations of _____ + _____ + _____ + _____. This less accurate estimate would be _____.



Estimations Revisited

Estimate the answers to each of the following problems.

1. Heidi ran 2.6 miles, 6.2 miles, 3.7 miles, and 4.5 miles when jogging last week. Approximate her total distance for the week and her average distance per day.

   total = ____________________
   average = ____________________

2. Jim is buying material for a project. He needs $9\frac{3}{8}$ yards of red material, $5\frac{1}{8}$ yards of green material, and $8\frac{5}{8}$ yards of orange material. Approximate the total yardage needed, then use that to compute the total cost, if each yard cost $2.98.

   total yards = ____________________
   cost = ____________________

3. The Snoots measured their living room for new rugs. It measured $14\frac{3}{8}$ ft by $19\frac{7}{8}$ ft and the cost of the rug they picked out was $1.98 per square ft. Estimate the area of the room and the cost of the rug.

   area = ____________________
   cost = ____________________

4. In one week Annette wrote checks for $45.89, $119.67, $37.09, and $216.45. Approximately how much did she spend that week? If her account had approximately $500 in it at the beginning of the week, estimate how much she had left by the end of the week?

   expense = ____________________
   balance = ____________________

5. Marvin buys a 15.6 lb turkey which costs $1.79 per lb. Approximate the cost of the bird. If he pays with a $50 bill , approximate his change. He is planning on having four people to dinner (including himself). Estimate how many lbs of turkey he is allowing for each guest?

   cost = ____________________
   change = ____________________
   lbs. = ____________________

Mental Math Challenge 3

Fill in the table below with the missing equivalent fraction (in lowest terms), decimal and/or percent. You will have ten minutes to complete this exercise.

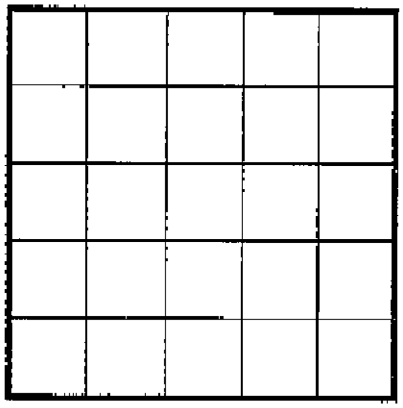
	Fraction	Decimal	Percent
1.			4%
2.			50%
3.	$\frac{1}{10}$		
4.		0.15	
5.			180%
6.			75%
7.		3.4	
8.	$\frac{7}{10}$		
9.	$\frac{1}{200}$		
10.		$0.\overline{6}$	
11.	$\frac{9}{20}$		
12.		0.2	

13.	$\frac{2}{5}$		
14.			150%
15.			30%
16.	$\frac{1}{4}$		
17.	$\frac{3}{200}$		
18.	$\frac{1}{3}$		
19.		0.6	
20.			120%
21.	$\frac{4}{5}$		
22.			400%
23.		1.0	
24.		0.005	
25.		0.28	

2.4 Approximations of Square Roots

Square - a figure, used in geometry, that is a rectangle with the length equal to the width

Example:



A **square** is the result of multiplying a number by itself.

Example: $5 \times 5 = 25$

$5^2 = 25$ is read as "five *squared* is twenty five."

25 is called the *square* of 5 .

We could think of it as the area (or number of units) of the square with side 5 units.

Square root of a given number - that value such that when it is multiplied by itself yields the given number

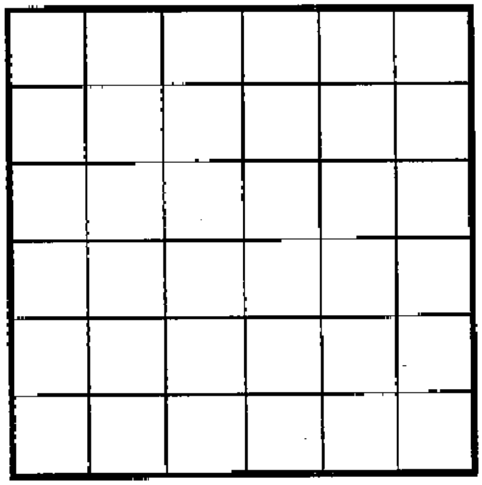
The *square root* of a number could be described as the "reverse of squaring". The *square root* could be thought of as the root or side from which the square was made. The symbol used for *square root* is $\sqrt{\ }$.

Example: $\sqrt{36} = \sqrt{6 \times 6} = \sqrt{6^2} = 6$

What number *squared* is equal to 36 ? The number 6 .

The *square root* of 36 is 6 .

Example:



$$6 \times 6 = 36$$

$$6^2 = 36$$

Hence, $\sqrt{36} = 6$

Example: $10 \times 10 = 100$, so $\sqrt{100} =$ _____.

Perfect Squares and Square Roots

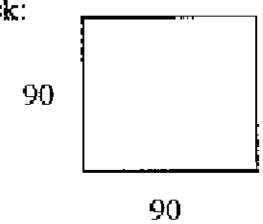
Simplify each of the following.

1. $\sqrt{25}$
2. $\sqrt{.0001}$
3. $(0.3)^2$
4. $\sqrt{8100}$
5. $\sqrt{.16}$
6. $(45)^2$
7. $\sqrt{64}$
8. $\sqrt{.0049}$
9. $(1.2)^2$
10. $\sqrt{36}$
11. $\sqrt{\frac{1}{4}}$
12. $\sqrt{2500}$
13. $(0.02)^2$
14. $\sqrt{.0004}$
15. $(3.1)^2$
16. $\sqrt{1.0}$
17. $(15)^2$
18. $\sqrt{.0064}$
19. $\sqrt{.09}$
20. $(0.1)^2$

Approximations of *square roots* can be introduced by first reviewing the previous discussion on *squares* and *square roots.*

Example: $\sqrt{8100}$, read "the square root of 8100" , means what number multiplied by itself is 8100 . It is equivalent to finding the side (root) of a square which has 8100 units. Hence, the square root or side is 90 .

To check:

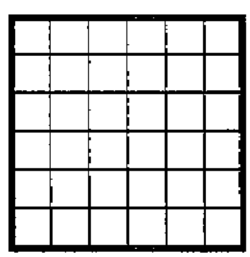


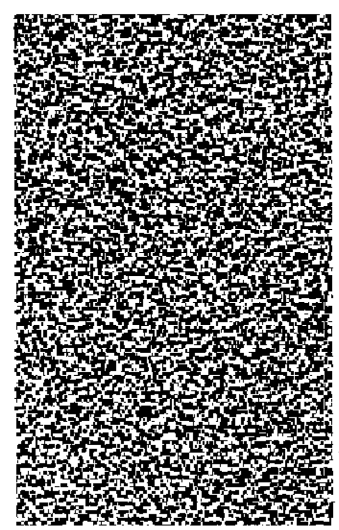
A square with side 90 will have an area of
90×90 or 8100 .

Example: Consider $\sqrt{40}$.

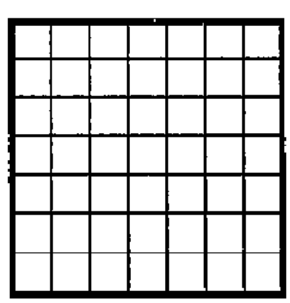
Can we draw a square with area 40 with a **whole number** side (root)?

If the side were 6 the area would be ______.





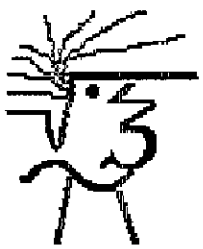
If the side were 7 the area would be ______



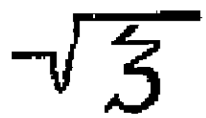
40 is **not** a **perfect square** but we can **approximate** its root by realizing that it falls between the whole numbers 6 and 7. The better approximation would be 6, since 40 is closer to the square of 6 than the square of 7.

Hence, $\sqrt{40} \approx 6$.

Square roots of numbers that are not perfect squares **cannot** be expressed as fractions. Therefore, these square roots **are not** rational numbers and are called *irrational numbers.* $\sqrt{3}$ is an irrational number, because 3 is not a perfect square. We can only approximate square roots of numbers that are not perfect squares. You may have noticed that Irrational Irv is made up of some irrational parts.



This is Irrational Irv.
He is usually irrational.



This is the square root of 3.
It is irrational.
$\sqrt{3} \approx 1.732$

This is the symbol for pi.
It is irrational.
$\pi \approx 3.14$

Approximation is an invaluable tool when answering multiple choice questions.

Example: $(1.2)^2$ = a.) 2.4 b.) 14.4 c.) 1.44 d.) 144

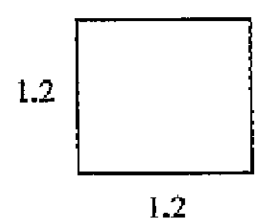
In this problem you are asked to find 1.2 *squared.*

The problem can be restated as:

Find the area of the square with side 1.2.

The area then is 1.2×1.2 or 1.44.

Hence, you would choose answer **c**.



If you don't exactly remember the rules for multiplication of decimals, reason that what you have is a square with a side a little more than 1. So, the area will be a little larger than a square with side 1. A square with side 1 has an area of 1. So, your square will have an area a little larger than 1. The decimal point, therefore, **must** be after the 1 to make any sense at all!

Square Roots

Choose the best answer for each of the following:

1. Approximate $\sqrt{2700}$

   a) 1100   b) 40   c) 500   d) 50

2. $\sqrt{0.64}$

   a) 0.32   b) 0.08   c) 0.008   d) 0.8

3. $\sqrt{1600}$

   a) 800   b) 400   c) 80   d) 40

4. $\sqrt{0.0016}$

   a) 0.0008   b) 0.004   c) 0.04   d) 0.08

5. Approximate $\sqrt{2400}$

   a) 1200   b) 50   c) 60   d) 500

6. $(0.12)^2$

   a) 0.24   b) 14.4   c) 1.44   d) 0.0144

7. $\sqrt{0.0049}$

   a) 7   b) 0.007   c) 0.07   d) 70

8. Approximate $\sqrt{500}$

   a) 250   b) 50   c) 22   d) 2500

9. $(4.3)^2$

   a) 184.9   b) 8.6   c) 18.49   d) 0.1849

10. Approximate $\sqrt{10,000}$

    a) 80   b) 90   c) 120   d) 5000

11. Approximate $(29.8)^2$

    a) 9000   b) 900   c) 60   d) 600

12. $\sqrt{1024}$

    a) 56   b) 512   c) 320   d) 32

2.5 Approximations of Percents

Understanding the concept of percent, which has previously been discussed, will provide a variety of approaches to solving problems containing percents.

Example: A taxable purchase is priced at \$80.45. The consumer wants to approximate the total cost which will include a 6% state sales tax.

$1\% = \frac{1}{100} = \frac{1}{10^2}$

As already noted, multiplication and division by powers of 10 can be easily completed by inspection. Therefore, when a percent is needed, division by a power of 10 can be a useful tool.

1% of $\$80.45 \approx \$.80$.

6% of $\$80.45 \approx 6 \times .80 \approx \$4.80$.

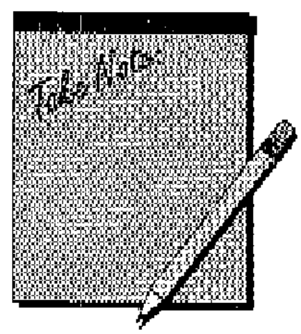
Knowing how to work with a few common percents is quite invaluable when dealing with everyday life situations. They are as follows:

100% of a number = all of the number, or 1 times the number

50% of a number = $\frac{1}{2}$ of the number, or the number divided by 2

25% of a number = $\frac{1}{4}$ of the number, or the number divided by 4

10% of a number = $\frac{1}{10}$ of the number, or the number divided by 10



Examples: 10% of 40 = 4 (40 divided by 10)

10% of 38 = 3.8 (3.8 divided by 10)

50% of 82 = 41 (82 divided by 2)

25% of \$120 = \$30 (120 divided by 4)

Find each of the following.

25% of 24

10% of 37.68

50% of \$288

It is relatively easy to find **20%** of a number. First find 10% of the number, then double it (multiply the answer by 2).

Example: Find 20% of 70 .

First find 10% of 70 , which is 7 .

Then, multiply this by 2 , which is 14 .

Hence, 20% of 70 = 14 .

Example: Find 20% of $32 .

10% of $32 is $3.20 .

So, 20% would be 2 × $3.20 .

Hence, 20% of $32 = $6.40 .

Being able to perform this operation **mentally** is quite helpful, especially when **tipping at a restaurant**!

It is also quite easy to find **150%** of a number. First find 100% of the number (all of it), then find 50% of the number (half of it), and then add the two results.

Example: Find 150% of 82.

First find 100% of 82 , which is 82 .

Next, find 50% of 82 , which is 41 .

Finally, add the two results.

Hence, 150% of 82 = 82 + 41 = 123 .

For those of you who prefer to only leave a **15%** tip, the folllowing example should prove quite useful. First find 10% of the number (the number divided by 10), then find 5% of the number (half of your first answer), and then add the two results.

Example: Find 15% of $20 .

10% would be $2 .

5% would be half of that result, or $1 .

Now, add these two answers.

Hence, 15% of $20 = $3 .

Find 15% , 20% , and 150% of 40 .

Of course, there will be many times when finding the exact answer to the question "What is XXX% of XXX is not practical, feasible, or desirable. When this is the case it's time to **estimate**! For example, let's say that you are in a restaurant and you want to figure out the amount of tip to leave. More than likely, the total amount of the bill will be a decimal amount. It would be most embarrassing to whip out a calculator to figure out the tip and embarrass yourself in front of either your date, your boss, or (worst yet) your mother-in-law! But have no fear---you **can** handle this gracefully!

Example: Approximate 15% of $27.68 .

In this case, first round the dollar amount to the nearest dollar. So, $27.68 would approximate $28 .

[Sometimes it may be easier to round to the nearest **ten** dollars.]

Then, figure the amount of tip mentally.

10% of $28 is $2.80 , and 5% of $28 is half of $2.80, or $1.40 .

So, 15% of $28 would be $2.80 + $1.40 or $4.20 .

Hence, you should leave approximately a **$4** tip.

Once again, if this seems to be too difficult, round to the nearest **ten** dollars. Finding 15% of **$30** mentally would be easier.

What amount of tip would you leave?

F.Y.I.

For all you pennypinching students
or those on a fixed income:
Tips should be figured on the total bill before tax

In the previous example, the estimation involved approximation of the given amount in the problem. At times, you may need to approximate the given **percent** in the problem to estimate the answer.

Example: *Estimate* 10.2% of 63 .

First, round the value preceding the % sign.

$10.2\% \approx 10\%$

Then find the answer using your knowledge of the common percents mentioned in this section. So,

10% of $63 =$ ______.

Hence, $10.2\% \approx 6.3$.

Understanding these relationships will help to determine both approximate answers **and** exact answers by calculating mentally.

Estimating with Percents

Use **approximations** to find **estimates** of each of these:

1. Find 25.8% of 4000.

2. What is 48.3% of 1600?

3. 9.8% of 1200 is what number?

4. Find 21% of 82.

5. What percent of 800 is 78?

6. 10.5% of what number is 420?

Choose from **1%** , **10%** or **100%**

7. 7 is ______________ of 70.

8. 15 is ______________ of 15.

9. 11 is ______________ of 1100.

10. 8.3 is ______________ of 83.

Find **exact** solutions to each these **mentally**.

11. 17 is 10% of ______________.

12. 8.5 is 100% of ______________.

13. 9 is 50% of ______________.

14. 7 is 25% of ______________.

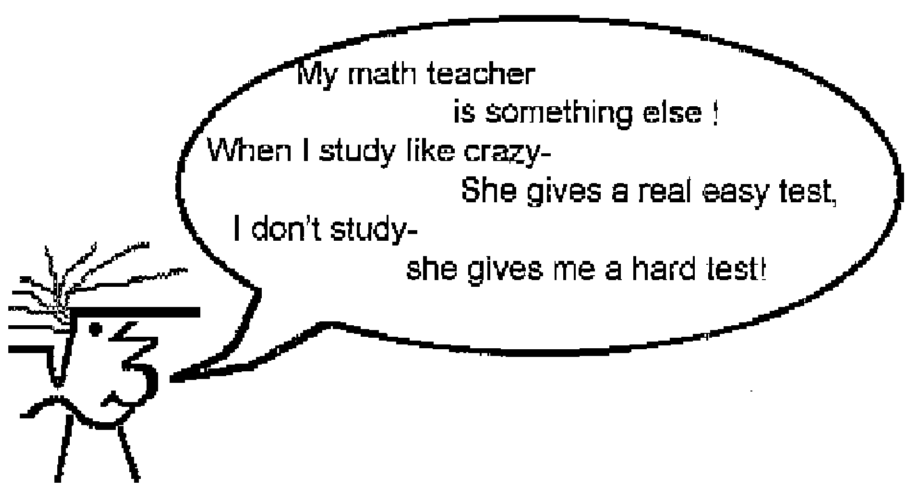
15. 20 is $33\frac{1}{3}$% of ______________.

2.6 Test Taking, Studying, and Problem Solving Strategies

It is common for people to become anxious when taking tests. Unfortunately, this anxiety often affects the results of the test, which then do not give an accurate measure of knowledge. Tests should be the tools used to determine how to approach the next level of learning. It is therefore especially important that part of the learning experience be devoted to test taking strategies.

The goal of this unit is to learn how to demonstrate what the test-taker knows by helping to minimize those common obstacles that usually distort test results. Learning and practicing *problem solving stategies, study stategies*, and finally, *test day strategies*, should acomplish this goal.

To begin your journey toward this goal, first rip out and complete the timed test on the following page.



* * * *

This is a timed test designed to see how well you can follow written instructions. Follow each step , in order. You will be given three minutes to complete this test.

1. Before doing anything else, read the following instructions carefully and completely.

2. Write your name and social security number in the lower right-hand corner.

3. Write your age in the upper right-hand corner.

4. Underline you last name only.

5. Tear off the number of stars in the left top corner that is the answer to 100 + 24 - 121

6. Turn the paper over and draw a right triangle.

7. Underline all of the nouns in step 3.

8. If your first name begins with a vowel, draw a circle around just your first name.

9. Draw a box around all the even numbers on this page.

10. Now that you are finished reading the instructions, only do steps one and two. Sit quietly and wait for everyone else to complete the test.

Directions for Scoring the Timed Test

If you don't already realize it, if you mutilated your page or did anything more than steps 1 and 2, you need to be more careful when following directions. Are you, by any chance, one of those people who assemble things ***before*** reading the directions? There is nothing inherently wrong with this approach, but in doing assignments or taking tests ***following directions exactly*** is crucial for success.

Next, fill out the Math Study Skills Evaluation and score.

Math Study Skills Evaluation

Place a check in the column that best describes how often each of the following takes place.

	seldom	sometimes	usually
Problem Solving			
1. I read the entire problem before starting.	____	____	____
2. If I can't think of where to start a problem, I feel upset.	____	____	____
3. Before beginning a problem, I ask myself what I am being asked to find.	____	____	____
4. I estimate my answers before starting the computations.	____	____	____
5. I check answers by rechecking my work.	____	____	____
6. If I have trouble with a problem, I mark it so I can ask about it later.	____	____	____
Studying for the Test			
7. To study for the test, I just read over problems, my notes, and the text.	____	____	____
8. I always study alone.	____	____	____
9. I study most the night before the test.	____	____	____
10. I re-do a lot of the homework and classwork problems.	____	____	____
11. I sometimes forget the rules and have to look back at them.	____	____	____

	seldom	*sometimes*	*usually*
12. I time myself when practicing.	____	____	____
13. I practice all of one kind of problem at the same time.	____	____	____
14. I find out as much as I can about the test format and how it will be graded.	____	____	____
15. I practice estimating my answers.	____	____	____

Taking the Test

	seldom	*sometimes*	*usually*
16. My mind goes blank when I take a test.	____	____	____
17. I picture myself doing well on the test.	____	____	____
18. I get nervous when I take a test.	____	____	____
19. If I come to a problem I can't do, I stick with it.	____	____	____
20. I check to see if I've answered the question asked.	____	____	____
21. If my answer is one of the choices, I don't bother checking.	____	____	____
22. I start with the first problem and work straight through.	____	____	____
23. I view a test as being given the opportunity to show what I know.	____	____	____
24. I use self-talk to guide myself through each problem.	____	____	____
25. I estimate my answers before I do each problem.	____	____	____

Scoring the Math Study Skills Evaluation

First look at the checks for the item numbers listed below. Give 2 points for each usually
1 point for each sometimes
0 points for each seldom

1 _____ point(s)
3 _____ point(s)
4 _____ point(s)
5 _____ point(s)
6 _____ point(s)
10 _____ point(s)
12 _____ point(s)
14 _____ point(s)
15 _____ point(s)
17 _____ point(s)
20 _____ point(s)
23 _____ point(s)
24 _____ point(s)
25 _____ point(s)

Group 1 total points________

Next, look at the checks for the remaining item numbers listed below. Give 2 points for each seldom
1 point for each sometimes
0 points for each usually

2 _____ point(s)
7 _____ point(s)
8 _____ point(s)
9 _____ point(s)
11 _____ point(s)
13 _____ point(s)
16 _____ point(s)
18 _____ point(s)
19 _____ point(s)
21 _____ point(s)
22 _____ point(s)

Group 2 total points________

Finally add the **total points for Groups 1 and 2** together and multiply the result by 2 to get your **score.**

Group 1 total points + Group 2 total points =____________ **Grand total points**
Grand total points × 2 = ____________ **your score**

If **your score** is above 85, you have excellent study and test-taking skills.
If **your score** is between 70 and 85, you have many good habits, but you can improve your skills.
If **your score** is below 70, you can greatly improve your skills

Dear Reader,

Hopefully, by now we have demonstrated your need for this particular topic. Becoming competent in math involves not only skill with computation, but understanding how to use your knowledge in real and test situations. There are three areas we will address.

Problem solving strategies is the first area of concern. As pointed out by the timed test on following directions, the very first step in solving any problem is to read the entire problem before attempting any activity. If you do this first you are less likely to make up your own problem or misinterpret what you are being asked to do. As mentioned before, beginning computations before knowing the exact goal may be misleading and a complete waste of time.

After you have read the problem through, ask yourself what you are being asked to find, writing this down if necessary. For example, you may be asked to find the total cost of a transaction or you may be asked to find only the tax. If you do all the computations correctly but fail to realize exactly what you are suppose to find for your final answer, all your work is in vain. Along these same lines, make sure you read exactly what the problem says and do not make up your own problem. If you are in the habit of finishing sentences for others, realize that this does not work when you are not intimate friends with the author of the math problem being considered.

Once you are certain of exactly what you are looking for, estimate your answer before doing any computations. Sometimes, especially in a multiple choice situation, estimating will actually lead to the correct answer without even doing the computations. Estimation <u>is not</u> guessing and <u>is</u> a valid and useful method for problem solving. Good estimation skills <u>alone</u> can greatly increase your success as a problem solver. Something you can do to prove this fact to yourself is to look back at some test, quiz or homework problems that you got wrong and see if an estimate would have indicated the error.

Only after you have completed all the preliminary steps, should you begin to actually work out the problem. When working on the problem, try to use your understanding of the concepts, instead of merely using a memorized rule. Use self talk to lead you through the steps, asking yourself appropriate questions and checking the reasonableness of each step. Make sure you talk positively to yourself, replacing negative phrases such as, "Oh,! No! I never could do word problems!", with something more appropriate like, "Now, first let me see exactly what the problem is asking."

After you have calculated your answer, check your answer by comparing it to your estimate. When you check a problem by re-doing it, if you have made an error, you will probably just make the same mistake again. This explains why many mistakes are not discovered. By checking with a previously made estimate, you can avoid needless mistakes and ultimately save time.

The final step in any problem should be to re-read the original problem along with your answer to be certain that the answer you have is indeed the answer to what was asked. This will catch careless errors you have made in transferring the problem to your paper (outright copying mistakes) along with any reading errors.

When doing an assignment, be sure to indicate those problems with which you need help. This will prepare you to ask specific questions concerning those problems. If you do not indicate those problems, you may not remember which ones you had questions about, or they may be hard or impossible for you to locate.

Study strategies is our second topic of concern. Studying math is *practicing problems*. Before doing any problems make sure you have the answers available. There is absolutely no value in just doing problems if you have no way to check for correctness. Make sure when you practice doing problems that you use the problem solving strategies we have already discussed, particularly checking the problems using your estimation skills.

Mix up the kinds of problems you practice. While you may be able to do 100 of the same type of problem quicker, it is most beneficial to vary the problems. Remembering how to do a particular kind of problem is what most people need to practice. Doing just one of each type will be very beneficial and a much wiser use of your limited time. (Using your study time for the maximum benefit is the goal). Along the same lines, make sure you limit your study time on the problems you have already mastered. Although it may benefit your ego, you really need time to practice those problems which are truly problems. (A really good way to mix up problems is to make flash cards. Yes, flash cards.!) Look back at quizzes, homework, and classwork for practice problems. Concentrate on the ones that have given you trouble. If possible, work with a buddy. Give each other problems to solve and check each others work.

When you are practicing, make sure you use positive self-talk. Your mind believes whatever you tell it to believe. When you tell yourself that you cannot do something, whether this is true or false your mind believes it, and it becomes a self-fulfilling prophesy. When you catch yourself being negative, just replace these thoughts. Ask yourself a question on how to best begin or remind yourself that you have indeed been successful thus far and you will be able to do the activity. When you speak positively to yourself it actually frees the mind to begin the problem.

Research has shown that if you have a certain amount of total time, studying frequently for short periods is more beneficial than longer, less frequent study. For that reason, you should try to space out your study time and avoid cramming. You should also time yourself when practicing and try, if possible, to practice the same time of day as the test will be given. This will especially help those of you who can always do the classwork and homework but freeze when given a test. Try to make your study situation as similar to the test situation as possible. For example, if you cannot eat during a test period, do not snack when practicing.

Make sure that before you begin preparing for a test, you find out about the test.

1. What is the test format? Will it be multiple choice, free response, etc.?

Make sure you practice these types of problems.

2. How will it be graded? Will there be partial credit? Will only the correct answers count? Will the number of wrong answers be subtracted from the number of correct answers, such as in the SAT tests? In this case, it would not be beneficial to guess.

CAUTION! Generally, it is advisable to make educated guesses when taking multiple choice tests. However, be certain that you are aware of the grading procedure before time.

3. How many problems? How much time?

You should time yourself when you practice, allowing yourself the same amount of time for each problem that you will have on the test. Being timed and never practicing this way is a major reason for people panicking on tests. An activity to identify and strengthen weak areas would be to allow yourself less time for the practice test. Errors will surface more readily when you are rushing. Concentrate on those errors.

Last, but certainly not least, are **Test taking strategies.** If you are using all the other strategies mentioned you are on the way to having good test taking skills. One of the most important things you can do, before taking a test, is to put yourself in the best mental frame. (Bear in mind none of these strategies will help if you have not practiced at all.) Use relaxation techniques such as deep-breathing. Picture yourself doing well. Most important of all, think of the test as showing what you CAN do.

Make sure you follow the directions given. DO NOT make up your own directions.

Before beginning the test, review with yourself time constraints and decide how much time to allow for each problem. Do the problems you know how to do first. After you have done the problems you are able to do, go back and work on the ones that were more difficult. If you cannot begin a problem, or cannot finish within your allotted time, mark the problem and come back to it later. For multiple choice tests, if you can rule out any answers do so, then come back to the problem later if you have more time. Do not go over your allotted time for any problem until you have tried all the test questions. Many students do poorly on a test because they spend too much time on one problem they cannot do and do not have time to finish the other problems they could do.

Make sure you use all your problem solving strategies during the test. Talk to yourself positively. You should be your own best fan and supporter.

Test Taking, Study, and Problem Solving Strategies Summary

Read through and *highlight* the important points in the letter on pages 90 though 94. Referring to what you've highlighted, fill in the summary below. Compare your summary with the summary on page 163.

Problem solving strategies

Study strategies

Test taking strategies

2.7 Using Approximations to Solve Application Problems

Applying your knowledge of percents, particularly the common percents introduced to you in section 2.5 , and the strategies stressed in section 2.6 , you will most likely find that "word" problems aren't so bad after all.

Example: Stacey found a dress that she liked on a "50% off---clearance" rack at the local boutique. The regular price of the dress was $64.95 . About how much would the dress cost her?

Since 50% of a number is half of it, find half of the regular price.

First approximate the regular price to be $64 since it is easier to find half of an even number mentally.

Hence, the dress would cost Stacey around _____.

Example: Danny complained to his dad that he had to pay $2 interest to his neighbor for borrowing some money at the rate of 10%. He said, "Gee, I didn't borrow *that* much off of him!" Just how much *did* he borrow?

Think to yourself, "$2 is 10% of what amount?"

In other words, 2 is $\frac{1}{10}$ of what number? _____

Hence, Danny must have borrowed _____.

Example: Kevin took his date, Lauren, out to dinner for her birthday. When the waiter came with the check, Kevin wasn't sure how much tip to leave. The bill total was $43.78 . He wasn't **overly** impressed with the service, so he decided to leave a 15% tip. What amount should he have left?

First round the amount to _____.

Next, find 10% or _____, then 5% or _____.

Hence, 15% of $43.78 ≈ $6.60 ≈ a $____ tip.

Percent Practice

For each of the following write

1%, 10%, 50%, 100%, and 150%

1. $200

2. 4200

3. $15.87

4. 65

5. 85,000

6. $24.00

Choose the correct answer using your knowledge of percents.

7. 50% of 360 is
   18 *180* *1800*

8. 80% of 65 is
   52 *520* *5200*

9. 40% of 2000 is
   8000 *800* *80*

10. 100% of 30 is
   30 *300* *3000*

11. 60% of 420 =
   25.2 *252* *2520*

One of the numerals in italic print in each of the following statements will make a true statement. Using your knowledge of percents circle the right choice.

12. 90% of ***6, 60, 600*** = 540

13. ***8%, 80%, 800%*** of 20 = 160

14. 240 = 8% of ***30, 300, 3000***

15. 18 = ***6%, 60%, 600%*** of 3

16. 150% of 640 = ***96, 960, 9600***

Choose the correct answer:

17. Roseanne bought a purse for $20. If the sales tax in her state was 4%, was the amount of the tax ***8 cents, 80 cents or 8 dollars*** ?

18. Mr. Smith has to pay federal income tax on $ 1800. The rate is 20%. Is his tax ***$3.60, $36, or $360?***

19. Glassboro High School has an enrollment of about 500 students. On a certain day, 95% of the students were present. Were there about ***4800, 480 or 48*** students presents?

Estimating With Problems Involving Percents

Try to **estimate** the answers using your knowledge of the following percents:
5%, 10%, 20%, 25%, 50%, 75%, and 100%

1. Find $8\frac{1}{2}\%$ of 600.

2. 45% of 8000 is what number?

3. 67 is 9.5% of what number?

4. Find 112% of 876.

5. What percent of 400 is 82?

6. What is 4.75% of 700?

7. 32 is 205% of what number?

8. Find 15% of $60.

Find **approximate** answers for each of the following application problems using estimation.

9. Lauren pays \$310 interest for a 1-year loan at 10.5%. About much money had she borrowed?

10. On a test, Cari had 85% of the problems correct. If she got 15 problems right, about how many questions were on the test?

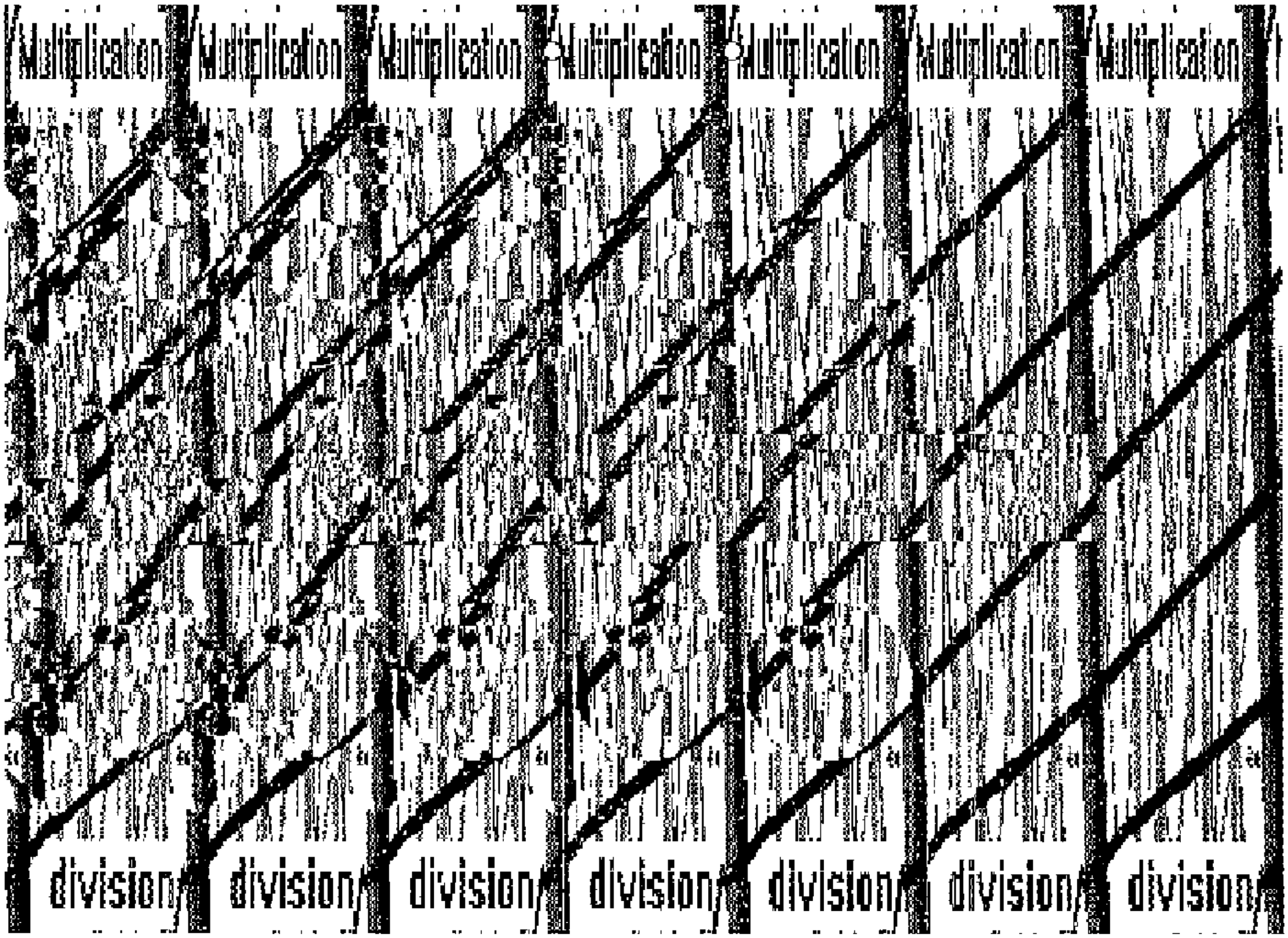
11. Walt has 27% of his pay withheld for various deductions. If he earns \$480 each week, about how much money is being withheld?

12. Christiana is taking a test on which she must receive a 74% to earn a B for the course. If the test contains 80 questions, about how many must she get right?

13. Carol must make a $9\frac{1}{2}\%$ down payment in order to buy a new car. If the sticker price of the car is \$12,000, about how much money would she have to put down?

14. Sales tax in a certain state is $5\frac{1}{2}\%$. If you intend to buy a TV that sells for \$640, about how much tax would you have to pay?

15. If Bob has \$7000 in a savings account, about how much interest would he earn in one year if the bank offers a rate of 4.65%?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Unit 3

3.1 Introduction and Skill Building Problems

To understand multiplication and division of rational numbers, it is necessary to recognize the many forms of rational numbers and the methods that may be used to complete their basic operations. Multiplication and division are called "inverse operations".

Example: (multiplication)

Find 3 groups of $(\times)$ 4.

$$3 \times 4 = 12$$

Example: (division)

How many groups of 4 are contained in 12 ?

$$? \times 4 = 12$$

$$? = \frac{12}{4} = 3$$

Multiplication rules are the same whether the rational number is in fraction form or decimal form since decimals are special fractions.

Example: To multiply $\frac{1}{2} \times \frac{5}{7}$, you would first multiply their numerators and then multiply their denominators.

So, $\frac{1}{2} \times \frac{5}{7} = \frac{1 \times 5}{2 \times 7} = \frac{5}{14}$.

Example: $\frac{7}{10} \times \frac{11}{100} = \frac{77}{1000}$

However, since $\frac{7}{10} = 0.7$, and $\frac{11}{100} = 0.11$, 0.7×0.11 should be equal to $\frac{77}{1000}$ also, but written in **decimal** form as 0.077.

This is precisely what the answer **is**.

0.7×0.11 **does** equal 0.077 because of the following steps:

Step 1: Multiply the 7 times the 11, (numerator times numerator).
This result is 77.

Step 2: Multiply 10 times 100, (denominator times denominator). This result is 1000. Common procedure for this is merely to count the decimal digits in the problem.

Step 3: Use the product from Step 2 to determine the position of the decimal in the product from Step 1.

Note that steps 1 and 2 for the *decimal multiplication* are exactly the same as those for the *fraction multiplication*!

Hence, the first **fraction** answer is equivalent to the second **decimal** answer since they both have the same value of 77 "thousandths".

Consider the following examples which show that *fraction multiplication* and *decimal multiplication* will result in equivalent forms of the **same** values.

Example: $2\frac{1}{2} \times \frac{2}{5} = \frac{5}{2} \times \frac{2}{5} = 1$

$2.5 \times 0.4 = 1.00 = 1$

Example: $2 \times \frac{3}{4} = \frac{2}{1} \times \frac{3}{4} = \frac{6}{4} = 1\frac{1}{2}$

$2 \times 0.75 = 1.50$

200% of $\frac{3}{4} = 2 \times \frac{3}{4} = \frac{2}{1} \times \frac{3}{4} = \frac{6}{4} = 1\frac{1}{2}$

[Remember that $200\% = 2$ and 'of' means multiply.]

200% of $0.75 = 2 \times 0.75 = 1.50$

Example: $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

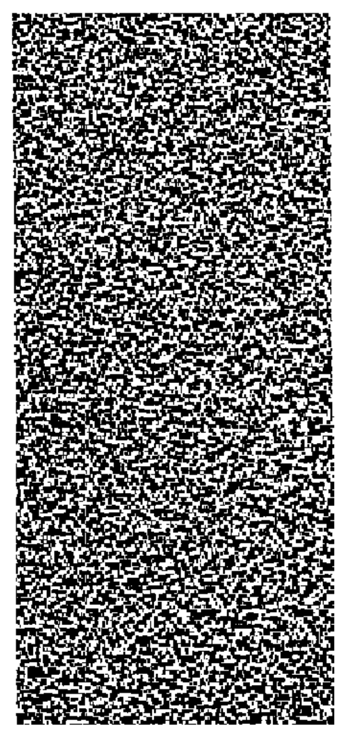
$0.5 \times 0.75 =$ _____

50% of $\frac{3}{4} =$ _____ $\times \frac{3}{4} =$ _____

50% of $0.75 =$ _____ $\times 0.75 =$ _____

Example: $\frac{1}{4}$ of $\frac{2}{5} = \frac{1}{4} \times \frac{2}{5} = \frac{2}{20} =$ _____

$0.25 \times 0.4 =$ _____



Consider the following examples showing the use of the word 'of' for multiplication.

Example: $\frac{1}{3}$ of $300 = \frac{1}{3} \times \frac{300}{1} = \frac{300}{3} = 100$

Example: 25% of $120 = \frac{1}{4} \times 120 =$ _____

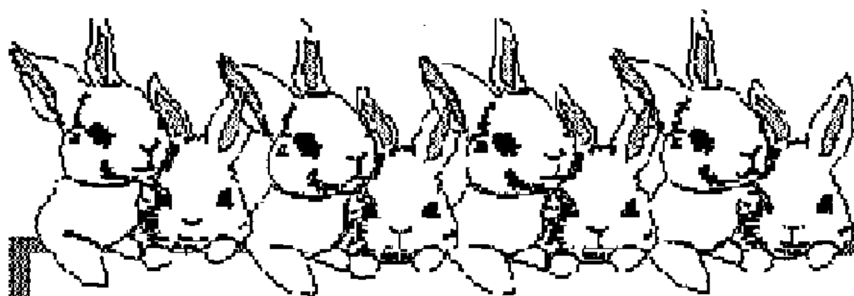
Example: To find 72% of 200, you could either work with the

72% in decimal form or in fraction form:

$$\frac{72}{100} \times \frac{200}{1} = 144$$

or

$$0.72 \times 200 = 144$$



We tend to think of multiplication as always making a quantity larger.

Don't forget, when multiplying by a number less than 1, the answer will be smaller!!!

Division

Choose the correct statement for each of the following expressions.

1.	$\frac{12}{6}$	a) 6 divided by 12	b) 12 divided by 6
2.	$3 \div 36$	a) 3 divided by 36	b) 36 divided by 3
3.	$9\overline{)18}$	a) 9 divided by 18	b) 18 divided by 9
4.	$\frac{3}{4}$	a) 3 divided by 4	b) 4 divided by 3
5.	$0.5 \div 3.7$	a) 0.5 divided by 3.7	b) 3.7 divided by 0.5
6.	$\frac{\frac{1}{2}}{5}$	a) $\frac{1}{2}$ divided by 5	b) 5 divided by $\frac{1}{2}$
7.	$4\overline{)1.6}$	a) 4 divided by 1.6	b) 1.6 divided by 4
8.	$\frac{15}{0.3}$	a) 15 divided by 0.3	b) 0.3 divided by 15
9.	$14\frac{1}{2} \div 7$	a) 7 divided by $14\frac{1}{2}$	b) $14\frac{1}{2}$ divided by 7

Choose a correct expression for the following :

10.	$\frac{1}{2}$ divided by $2\frac{1}{3}$	a) $\frac{1}{2} \div 2\frac{1}{3}$	b) $2\frac{1}{3} \div \frac{1}{2}$
11.	623 divided by 1002	a) $\frac{623}{1002}$	b) $\frac{1002}{623}$
12.	0.76 divided by 0.026	a) $0.76\overline{)0.026}$	b) $0.026\overline{)0.76}$
13.	8 divided by 16	a) $\frac{8}{16}$	b) $\frac{16}{8}$
14.	3 divided by $\frac{1}{2}$	a) $\frac{1}{2} \div 3$	b) $3 \div \frac{1}{2}$

Choose an equivalent expression for each of the following:

15.	$0.5 \div 3.7$	a) $0.5\overline{)3.7}$	b) $3.7\overline{)0.5}$
16.	$\frac{\frac{1}{2}}{5}$	a) $\frac{1}{2} \div 5$	b) $5 \div \frac{1}{2}$

Reciprocal - a number that when multiplied by a given number results in 1

Examples: 4 is the reciprocal of $\frac{1}{4}$ since $4 \times \frac{1}{4} = 1$

$\frac{2}{3}$ is the reciprocal of $\frac{3}{2}$ since $\frac{2}{3} \times \frac{3}{2} = 1$

In general, *dividing* by a number is equivalent to *multiplying by its reciprocal.*

Example: $\frac{1}{5}$ of a number means dividing it by 5, so $\frac{1}{5}$ of 25 means $25 \div 5$.

Therefore, $25 \div 5 = \frac{1}{5}$ of 25

$= \frac{1}{5} \times 25$

$= 25 \times \frac{1}{5}$

Then, $25 \div 5 = 25 \times \frac{1}{5}$.

Hence, dividing by 5 means multiplying by $\frac{1}{5}$, its reciprocal!

Now, if you followed that ----you deserve a medal!

When *dividing fractions* you first change the problem into an equivalent multiplication problem by replacing the divisor (the second number) with its reciprocal before multiplying.

Example: $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{10}{12} = \frac{5}{6}$

Example: $\frac{3}{4} \div 6 = \frac{3}{4} \times \frac{1}{6} = \frac{3}{24} = \frac{1}{8}$

Example: $3\frac{1}{2} \div \frac{1}{4} = \frac{7}{2} \times \frac{4}{1} = \frac{28}{2} = 14$

Multiplying a number by 1, or a number equivalent to 1, doesn't change its value. Hence, multiplying the numerator and denominator of a fraction by the same number results in an equivalent fraction.

When *dividing decimals* care must be taken if the divisor is itself a decimal.

Example: To divide 2.05 by 0.5 consider the following explanation:

$$0.5\overline{)2.05} = \frac{2.05}{0.5}$$

$$\frac{2.05}{0.5} \times \frac{10}{10} = \frac{2.05 \times 10}{0.5 \times 10} = \frac{20.5}{5} = 5\overline{)20.5} = 4.1$$

Hence, to solve $0.5\overline{)2.05}$, first change it to $5\overline{)20.5}$.

Multiplication and Division of Rational Numbers

Perform each indicated operation.

1. 1.34×2.5

2. $\frac{2}{5} \div \frac{1}{2}$

3. $3.9 \times \frac{1}{3}$

4. $5.6 \div 0.2$

5. 2.4% of 0.3

6. $6 \div \frac{3}{4}$

7. $2\frac{4}{5} \times \frac{1}{5}$

8. $5.6 \div \frac{1}{8}$

9. $\frac{1}{3} \times \frac{2}{3}$

10. 35% of 2.04

11. $0.3\overline{)324}$

12. $\left(\frac{2}{3}\right)^2$

13. $60 \div 1.5$

14. $\frac{3}{4} \times 4$

15. $(6.1)^2$

16. 4×1.06

17. 0.05×4.21

18. $\left(\frac{4}{5}\right)^2$

19. 1.2% of 6

20. $1.3 \times \frac{1}{5}$

21. $\frac{38.2}{0.02}$

22. $3\frac{1}{2} \times 2.6$

23. 10% of 82.4

24. $4\frac{3}{4} \div 2$

3.2 Basic Application Problems

Let's reflect on "why" it is necessary to know how to perform basic operations with rational numbers.

Everyday, in a variety of ways, it becomes important for people to know how to use rational numbers.

All of us are consumers who make purchases and are involved in transactions that involve money. As responsible citizens, we all pay taxes. Our paychecks reflect routine deductions based on specific percents of out salaries.

Much of the news we are exposed to requires a knowledge of basic math to fully understand the world in which we live. We hear about the "Prime rate", tariffs on luxury cars imported from Japan, and studies about foods, medicines, death rates, car insurance, mortgages, loans, etc.

When we travel, we use maps and scales showing distances and sometimes changes in currency and time. The list is infinite. Briefly stated,

WE CANNOT FUNCTION INDEPENDENTLY

IF WE DON'T HAVE SOME

BASIC KNOWLEDGE OF

MATH CONCEPTS.

Consider the following problem:

Turkey is on sale for $1.19 per pound. How much would $2\frac{3}{4}$ pounds cost?

First, ask yourself "What am I being asked to find?" ____________________

If you are not sure how to find this answer, try asking yourself if you could do a problem of the same type with easier whole numbers. An example of this problem could be

Turkey is on sale for *$1* a pound . How much would *3* pounds cost?

The solution to *this* problem is easier to see. The answer is $_______.

Next, think of the step or steps you followed to get the answer to your made up problem.

$\underline{\$1}$ per lb × $\underline{3}$ lbs.

Do the same step for the actual quantities in the problem.

$_____ per pound × _____lbs = $_________.

Notice that there is a decimal and a fraction in the computation. You could use either form to work the problem out. However, because the answer is *money*, using the decimal forms would make more sense.

Notice that the problem we made up is also a good estimate of the answer we should get.

Last, compare your answer to the estimate.

Word Problems Using Multiplication and Division

Solve each of the following. Remember to use the suggested steps for problem solving.

1. Turkey is on sale for \$1.19 per pound. How much would $2\frac{3}{4}$ pounds cost? If there are three people sharing the cost, how much should each one contribute?

2. If one quart of oil cost \$1.39, how much would 12 quarts cost? If a case (12 quarts) is on sale for \$16.95, would purchasing a case be a "deal"?

3. Jane's aunt sent her $7\frac{3}{4}$ lbs of cookies, which she decided to share with her 3 roommates. How much would each person get? (Assume the cookies are *equally* shared)

4. What is the average speed (velocity in miles/hour) of Lani, if she travels 63 miles in 2 hours and 20 minutes? Would you expect to find out she was traveling on foot, by car, or by plane?

5. A train averages 78.3 miles per hour. If Ted gets on the train at 2 PM, how far could he expect to travel by 7:30 PM?

One of the most important ideas that we have conveyed throughout our math discussion is that there are many different correct methods to solve applications. Learning gives us the freedom to select the method we prefer! One of the commonly preferred methods in specific applications involves the use of ratio and proportion properties as we discussed back in Section 1.4 . At that time you worked on a problem that involved the use of a map that was drawn to "scale". Let's review with a similar problem.

If an inch is used to represent 10 miles on the map and our trip measures $\approx$ 4" , find the actual miles we will travel.

Step 1: Set up a proportion.

$$\frac{\text{1 inch}}{\text{10 miles}} = \frac{\text{our trip in 'inches' on the map}}{\text{the actual 'miles' we will travel}}$$

Thus, $\dfrac{1''}{\text{10 miles}} = \dfrac{4''}{x \text{ miles}}$ or simply $\dfrac{1}{10} = \dfrac{4}{x}$.

Step 2: Solve the proportion for the missing term.

$$1 \times x = 10 \times 4$$

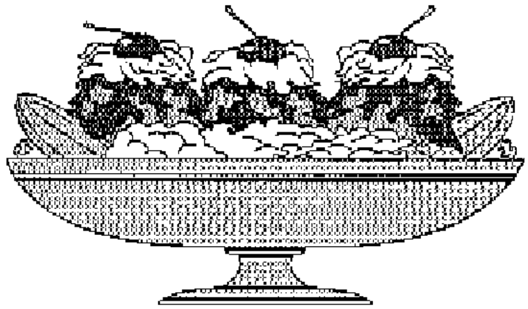
$$x = 40$$

Hence, we will actually travel 40 miles on our trip.

This review of solving application problems using ratios and proportions will prove quite useful in **solving percent problems** in the remaining sections of this book.

Ratios Revisited

Use the following information to set up the appropriate ratios. Do not reduce.



In this banana split one cherry has 8 calories,
one banana has 100 calories,
and the whole thing has 800 calories.

Write the ratios of: cherries to bananas,

bananas to cherries,

calories in a banana to calories in a cherry,

and calories in all the cherries to total calories.

Consider this group of cheering fans. Write the following ratios:

number of pennants to number of fans

number of fans wearing glasses to the number of pennants

number of fans with hats to those without hats

Proportion Practice

For each of the given proportions, determine the value of x.

1. $5x = 100$

2. $\frac{1}{4}x = 40$

3. $\frac{x}{4} = \frac{12}{24}$

4. $\frac{x}{15} = \frac{1}{2}$

5. $\frac{\frac{1}{2}}{10} = \frac{x}{20}$

6. $\frac{9}{x} = \frac{\frac{1}{2}}{36}$

7. $\frac{\frac{1}{5}}{70} = \frac{x}{100}$

8. $\frac{\frac{2}{3}}{\frac{1}{5}} = \frac{25}{x}$

9. $\frac{x}{18} = \frac{\frac{2}{3}}{6}$

10. $\frac{\frac{1}{500}}{\frac{1}{200}} = \frac{x}{2000}$

Proportion Word Problems

Before doing the following problems you may want to review solving proportions. Make sure there is a unit match between the ratios. Remember, there are always 4 correct ways and 4 incorrect ways to set up any proportion problem. Don't forget to approximate your answers before beginning each problem.

1. Bobby spent $75.50 in 5 weeks (for snacks). Project how much he would spend on snacks in one year?

2. If a recipe call for $2\frac{1}{2}$ cups of flour for every 3 cups of blueberries, how many cups of flour would you need for 2 cups of blueberries?

3. To prevent AIDS, contaminated items should be washed with a solution of $\frac{1}{4}$ cup bleach in 1 gallon of water. If your bucket will only hold 1 quart of water, how much bleach should you use?

4. If $2\frac{3}{4}$ lbs of grapes cost Jim $4.32, how much would 2 lbs cost?

   What is the price per pound?

5. Jean's car went 253 miles on $10\frac{1}{2}$ gallons of gas. If she fills her 14 gallon tank, what is the furthest she can expect to travel before refilling her tank?

3.3 One-Step Percent Problems

If all values were first divided into one hundred equal parts, then each part would be one percent of the whole quantity. It might be easy to think about the word "percent" as the phrase "per cent" as it was once used, in another way. Perhaps the relationship of a dollar and a penny would help to remind us that if a product that was easily and usably divided into 100 equal parts, and cost one dollar, how much would we get per cent (for a penny). Realistically, in today's world, we probably could get next to nothing per cent, but the thought might help to make "percent" more clear.

Picture a giant bag of popcorn that costs one dollar. If we were dividing the popcorn equally into one hundred small bags, each bag would cost one penny, and that portion would be 1%, or $\frac{1}{100}$, or one of the hundred equal parts.

Comparing how any portion (percentage) of a value, compares to the whole quantity (base) had it been divided into 100 equal parts, translates the ratio into the language of "percent".

Example: How does 6 of 200 parts translate into the language of "percent"?

$$\frac{?}{100} = \frac{6}{200}$$ or "What number is to 100 as 6 is to 200?"

(Think of our previous popcorn example just to reinforce our approach to percent.)
If 200 is the entire quantity, then divided into 100 equal parts, there would be 2 in each part. If 200 cost one dollar, then we could get 2 for a penny, or " 2 is 1% of 200 ".

How many groups of 2 are contained in 6 ? Three.

Therefore, " 6 is 3% of 200 " or $\frac{3}{100} = \frac{6}{200}$.

Although not all problems provide values that are as obvious as the example, the thinking is the same.

For the statement 20 is 40% of 50 , the percent is 40 , the whole (*base*) is 50 , and the remaining part (*percentage*) is 20. If the percent is written in fraction form, these values can be expressed in a "percent proportion" as follows: $\frac{40}{100} = \frac{20}{50}$.

In general terms, a "percent proportion" would be $\frac{\text{percent}}{100} = \frac{\text{part } (\textit{percentage})}{\text{whole } (\textit{base})}$.

Remember, in one ratio the 100 would always be the term under the percent value since it is the denominator for the percent written in fraction form. To determine where to place the other terms, keep in mind that the whole (*base*) will most often follow the word "of". Once you have determined the percent and the whole in a problem, there is only one quantity left and only one place left to put it in the proportion! Since in any percent problem we would always know two of the three quantities, we could always solve for the missing, unknown term. Problems solved by this ***proportion method*** follow.

Example: 12 is 48% of ?

The percent is the easiest value to pick out, so place that into the proportion first. $\frac{48}{100} = \frac{?}{?}$

Next, look for the whole (*base*). This value is the next easiest to spot since it is most often after "of". In this problem it is missing or unknown, so call it "x".

$$\frac{48}{100} = \frac{?}{x}$$

Finally, there is only one value left in the problem, the 12 , and only one place left in the proportion.

$$\frac{48}{100} = \frac{12}{x}$$

You are now ready to solve for the missing term in the proportion.

Since $x = 25$, then 12 is 48% of 25.

Example: ?% of 60 is 20 .

The percent would be x (since it is unknown), the whole (*base*) is _____, and the part (*percentage*) must be _____.

$$\frac{x}{100} = \frac{20}{60}$$

Since $x = 33.\overline{3}$ or $33\frac{1}{3}$, then $33\frac{1}{3}\%$ of 60 is 20 .

Example: ? is 30% of 150 .

The percent is _____, the whole (*base*) is _____, and the part (*percentage*) is _____.

$$\frac{30}{100} = \frac{x}{150}$$

Since $x = 45$, then 45 is 30% of 150.

This last problem could also have been solved by a different method, one that has already been used. To find 30% of 150, you could have first expressed 30% in decimal form as 0.3. Then, remembering that "of" means **multiply**, you could have completed the solution by multiplying the 0.3 and 150. Hence, 30% **of** 150 means 0.3×150 or 45. Keep this in mind whenever you determine that the percent and the whole or base are **given** values in a problem. If this is the case, you can always use this alternate method. As you may have realized, this method is the quickest one if you are using a calculator.

Percent "One-Liners"

1. Find 25% of 240.

2. 16 is what percent of 80?

3. 827 is 10% of what number?

4. ____% of 400 is 72.

5. 34 is $8\frac{1}{2}$% of what number?

6. 7% of what number is 42?

7. Find 75% of 120.

8. 6.5% of ____ is 325.

9. $10\frac{1}{2}$% of what number is 420?

10. What percent of 180 is 120?

11. 65 is 30% of what number?

12. Find 110% of 82.

13. 40% of what number is 200?

14. 40 is what percent of 120?

15. What is 125% of 300?

16. 51 is ____% of 850?

17. What is $11\frac{3}{4}$% of 6000.

18. Find 150% of 2000.

19. 500 is what percent of 1500?

20. Find 9.8% of 1200.

Using your knowledge of percents, choose the correct answer for each of the following.

21. ____% of 500 is 200. *(4, 40, 400)*

22. 18% of ____ is 72. *(4, 40, 400)*

23. ____ is 30% of 150. *(4.5, 45, 450)*

24. 92 is ____% of 80. *(1.15, 11.5, 115)*

3.4 Application Problems Involving Percents

The significant aspect of having a tool is knowing how to use it. Such is also the case with the basic math concepts we discuss in this book. Learning to use a tool requires practice, and true understanding of percent also requires practice.

Some practical applications of percent involve **commission, sales tax,** and **simple interest**. An example of each of these follows.

Example: Gary sells cars at a dealership that offers a 4% rate of commission. This means that for each car that he sells, he receives a portion (*percentage*) of the selling price which is called the ***commission.*** What commission would he receive if he sells a car for $12,000 ?

Percent "one-liner" : Find 4% of 12,000

Estimate: 10% is 1200 , so 5% is 600 .

Thus, the estimate would be less than $600.

Solution: Using the "percent proportion"

$$\frac{\text{percent}}{100} = \frac{\text{part } (\textit{percentage})}{\text{whole } (\textit{base})},$$

the *percent* would be the **rate of commission,**

the *whole (base)* would be the **total sales amount**, and

the *part (percentage)* would be the **commission.**

Thus, the modified proportion would be

$$\frac{\text{rate of commission}}{100} = \frac{\text{commission}}{\text{total sales}}$$

So, $\dfrac{4}{100} = \dfrac{x}{12000}$.

Hence, x = _____, his *commission.*

Once again, since the percent and the base are given, you could solve this problem by converting the percent to decimal form and then multiply it times the base.

$$12{,}000 \times 0.04 = 480$$

Example: Sara went to the sporting goods store to buy a new soccer ball. The cost of the ball was \$12 and she had to pay a ***sales tax*** of \$ .66 . What was the sales tax rate in her state?

Percent "one-liner" : What percent of 15 is 0.66 ?

Solution: The *percent* would be the **sales tax rate,**

the *whole (base)* would be the **total sales amount,**

and the *part (percentage)* would be the **amount of tax.**

$$\frac{\text{tax rate}}{100} = \frac{\text{amount of tax}}{\text{total sales}}$$

So, write the proportion $\frac{x}{100} = \frac{.66}{12}$.

Then $12x = 66$, and $x = 5.5$.

Hence, the *sales tax* rate was 5.5% .

Example: Jesse took out a loan for a year and had to pay \$360 interest. If the ***interest rate*** for the loan was 9% , how much money did he borrow?

Percent "one-liner": 360 is 9% of what number?

Solution: The *percent* would be the **interest rate,**

the *whole (base)* would be the **amount of the loan,**

and the *part (percentage)* would be the **amount of interest.**

$$\frac{\text{interest rate}}{100} = \frac{\text{amount of interest}}{\text{amount of the loan}}$$

So, write the proportion $\frac{9}{100} = \frac{?}{?}$.

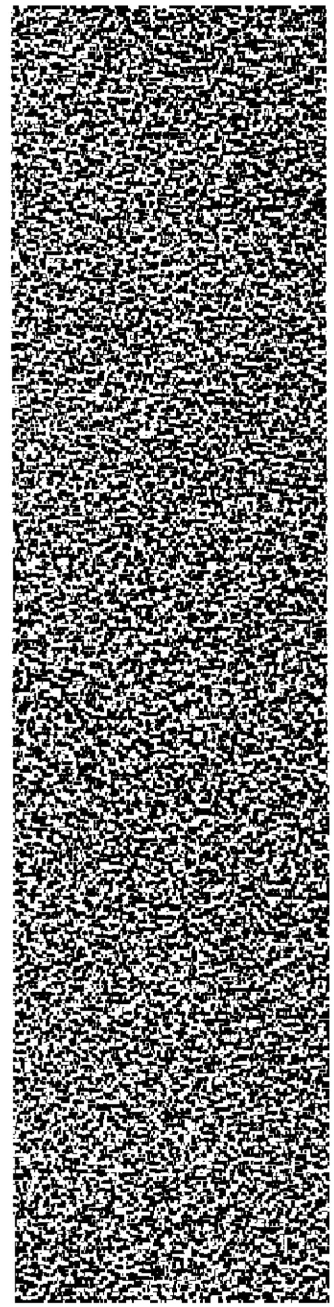
Then $9x =$ ______, and $x =$ ______.

Hence, Jesse borrowed ______.

Applications of Percents

Sales tax, Interest, Commission

Solve each of the following problems by first trying to estimate the answers, and then finding the actual answer by using the proportion method or a short-cut method.



1. How much sales tax would Danny pay on a purchase of \$28, if the tax rate in his state is 5%?

2. Joan works for Sunshine Realty where she receives a 6% rate of commission. If she is the agent for a house that sells for \$120,000, how much would she make on the sale?

3. Walt pays \$425 interest for a 1-year loan at a rate of 10.5%. Find the amount of his loan.

4. If Joey has to pay a $3\frac{1}{2}$% sales tax in his state and the amount of tax that he paid on a purchase of a new VCR was \$7, what was the price of the VCR?

5. The Waltons have to make an $8\frac{1}{2}$% down payment in order to buy a new car. If the car costs \$12,550, how much must they put down?

6. Margie works for 5% commission at the downtown boutique. If she wants to make \$1500 a month, how much merchandise would she have to sell?

7. Lori made \$5100 on the sale of an \$85,000 home. What was her rate of commission?

Percent problems may be handled using proportions. Consider the following problem.

If only the top 70% of the students in a mechanical drawing class pass for the year, how many, in a class of 24, would fail?

First, change 70% to the fraction or ratio that it represents.

70% = ________

This means that 70 out of each hundred students will pass the course.

Next use this ratio to set up the proportion

$$\frac{70 \text{ (pass)}}{100 \text{ (total in class)}} = \frac{x \text{ (unknown number passing)}}{24 \text{ (total in class)}}$$

Solve this proportion. x = ________. Because we can not have a part of a person, this number must be rounded to __________ so ______ students pass. Is this the answer to the question? _____

Refer back to the original question. How would you find the correct answer?

The correct answer is ______ students will fail. Check to make sure your answer makes sense.

Another way to approach the same problem is to begin by realizing that if 70% pass, then _______% fail. Following the same procedure as above, 30% means 30 out of 100 students.

$$\frac{30 \text{ (fail)}}{100 \text{ (total in class)}} = \frac{x \text{ (unknown number failing)}}{24 \text{ (total in class)}}$$

When we solve this proportion x = _________. Here again, we cannot have a part of a person, so we round off. The answer is __________ students fail. Make sure you check this .

There are many other ways to do this problem. Can you think of another way?

Refreshing Word Problems

Remember to approximate your answers *before* actually working out the problem.

Remember to practice checking each problem with the words of the *original* problem.

Remember to use self-talk if you can't get started.

1. Louise receives 11% of any overpayments made in error to the electric company which she can re-coup for her company. How much compensation would she receive for finding an error of $23,423?

   estimation ________ exact answer ________

2. If 232 people out to 348 surveyed prefer the taste of Zesty Zesto's, fill in the following ad.

   ________***% of those surveyed prefer ZESTO'S***

3. If 2 out of three people surveyed preferred Zesty Zesto's, fill in the following ad.

   ________***% of those surveyed prefer ZESTO'S***

4. Compare the answers in 2 and 3. When a percent is given, what information *is not* given? What should a wise consumer always ask about statements with %'s ?

5. If 232 people out to 348 surveyed prefer the taste of Zesty Zesto's, what fraction did not prefer them? Does this reduced form of the ratio give the reader any indication of how many people were surveyed?

6. If in a typical Math 010 class, 92% of the students improve their math scores greatly, in a class of 27, how many would you expect to improve?

7. Lily's allowance was increased by $3. If she received $12 prior to the increase, by what percent was her allowance increased?

8. Lily's younger brother, Nelson, got wind of the increase and requested his allowance be increased proportionally. If he is getting $8.00 now, by how much will his allowance increase?

9. Lily's wise mother approved the increase, with the condition that her children spend more time doing chores. If Nelson did 3 hours worth of work for an allowance of $8.00, how much time should he spend after his increase?

10. In a recent poll, 18% agreed with the proposed school budget. If the only choices were agree or disagree and 54 people agreed, how many people were polled? In a town of 3000, what % were polled?

Percent Practice 2

For each of the following write

10%, 50%, 60%, 40%, and 140%

1. $100

2. 5300

3. $18.00

4. 68

5. 80,000

6. $2.50

Choose the correct answer using your knowledge of percents.

7. 92% of 75 is
   6900 *690* *69*

8. 65% of 400 is
   26 *260* *2600*

9. 6% of 700 is
   42 *420* *4200*

10. 150% of 642 is
    9.63 *96.3* *963*

11. 15% of 880 =
    13.2 *132* *1320*

One of the numerals in italic print in each of the following statements will make a true statement. Using your knowledge of percents circle the right choice.

12. 46% of ***5, 50, 500*** = 23

13. ***4%, 40%, 400%*** of 650 = 260

14. 210 = 7% of ***30, 300, 3000***

15. 18 = ***6%, 60%, 600%*** of 30

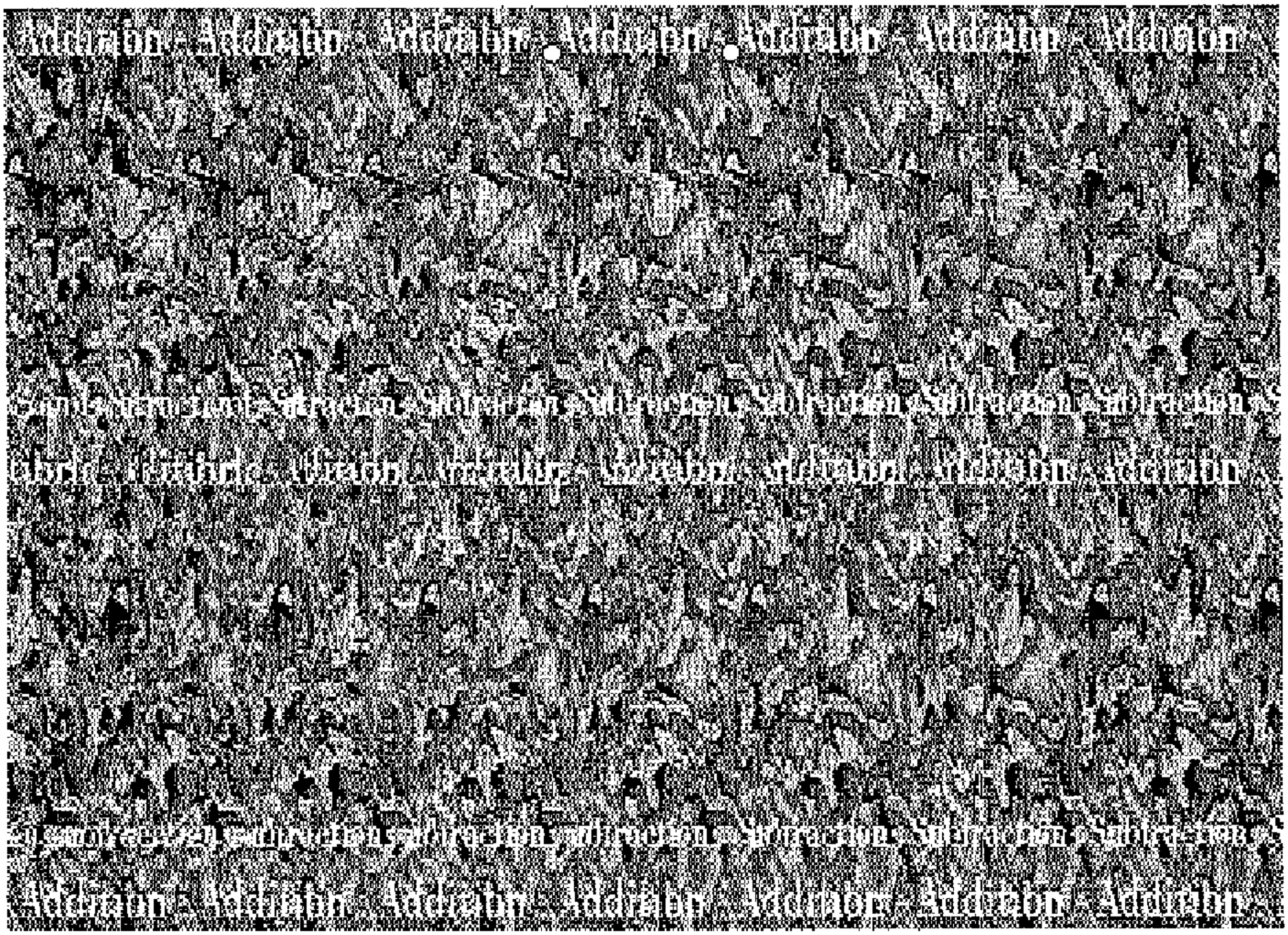
16. 15% of 640 = ***96, 960, 9600***

Choose the correct answer:

17. Mary answered correctly 16 of 20 problems on a test. Was her grade ***8%, 80%, 800%***?

18. Mr. Smith has to pay federal income tax of $1800. The rate is 20%. Was his income ***$900, $9000, or $90,000?***

19. Preppy High School has an enrollment of about 500 students. On a certain snowy day, 55% of the students were present. Were there about ***2800, 280 or 28*** students present?



Unit 4

4.1 Introduction, Skill Building, and Order of Operations

I suppose you were beginning to think that we'd never get to the point where we will use addition and subtraction. Let's begin by pointing out some things of which we are all aware. If I have 6 oranges and each costs 30 cents, I could use the multiplication operation to find that altogether they would cost $1.80. We also know that we may multiply **any** two fractions by multiplying the numerators and multiplying the *denominators*. We can multiply **any** two decimals. We can multiply **any** two quantities.

However, if I take the same 30 cents and 6 oranges, I cannot add them. We can only add (or subtract) the same *kinds* of things, the same *units*. Sometimes this is referred to as adding "like" or "similar" terms. The term "similar" can be misleading because we really mean ***exactly the same kind*** of expressions. The term that truly identifies "like" or "similar" expressions is **commensurate**. Commensurate means having a common measure. Therefore, when we add or subtract expressions they must have a common measure. For example, a term that identifies an amount of money, $1.50 (one dollar and fifty cents), cannot be added to or subtracted from a term that identifies a distance, 6.5 miles.

Common sense could help us avoid mistakes by recognizing that values that are not "commensurate" cannot be added or subtracted. If we tried to add the money term and the distance term in the example above, what would the answer represent? What label would we give our answer?

Now the fact that only oranges and oranges may be added may not seem particularly insightful, but it stops us from questioning why and when we need to use common denominators or line up the decimal points.

When one adds or subtracts, only the same units can be combined.

Example: To add the numbers $23 + 167$, most of us would rewrite the problem as

$$\begin{array}{r} 167 \\ +\ \ 23 \\ \hline \end{array}$$

We add the $7 + 3$ first because both of those numbers represent "ones". We now have 10 "ones" which is really 1 "ten" and 0 "ones". We then proceed to place a 0 under the "ones" column and "carry" the 1 over to the "tens" column with the 6 and the 2. And so on, and so on, and so on . . .

This whole column thing becomes very intuitive for us and is easily extended to decimal problems.

Example: To add 167.2 + 23.82 , we would rewrite it as

$$\begin{array}{r} 167.2 \\ \underline{+\ 23.82} \end{array}$$ again lining up the units to make it easier to add the "same" terms.

We do **not** need to line up the decimal points in a **multiplication** problem because we do **not** need to have like units!

Fractions follow the same format (or procedure) but look a lot more difficult. Again **we may only add like units**. Recall that for whole numbers we added the "ones" plus the "ones", etc., and for decimals we added the "tenths" plus the "tenths", etc. Both of these groups are easy to work with because they are based on powers of ten. So, when we get ten of any unit, we have one of the next higher unit. Fractions, on the other hand, are **not** all based on ten. When we apply our requirement of "commensurate" to fractions, the common measure now becomes the denominators of the fractions to be added or subtracted. Going a step further, the label or denominator of each fraction must be the same for addition or subtraction to occur, and the denominator of the result **must** be the same as those fractions involved in the operation. So, for instance, if we have 6 "eighths" and 4 "eighths", following the same train of thought as decimals and whole numbers, we now have 10 "eighths". Unfortunately, this **does not** equal one of any group, but instead $\frac{10}{8} = 1$, with $\frac{2}{8}$ left which reduces to $1\frac{1}{4}$. We must be careful **not** to add the denominators. If we read the problem to ourselves this mistake can be avoided.

Example: $\frac{2}{3} + \frac{2}{3}$ could be read as 2 "thirds" plus 2 "thirds".

After adding, we would then have 4 "thirds" or $\frac{4}{3}$ which is equivalent to $1\frac{1}{3}$.

If the denominator of the fractions are **not** alike, change the appearance but not the value of the fraction by finding a common denominator.

Let's first consider a *decimal fraction* to illustrate.

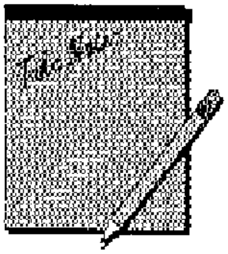
Example: To add 0.1 + 0.03 , which is the same as $\frac{1}{10} + \frac{3}{100}$, we would write it as

$$\begin{array}{r} 0.1 \\ \underline{+\ 0.03} \end{array} \quad \text{or} \quad \begin{array}{r} 0.10 \\ \underline{+\ 0.03} \end{array}$$, which is the same as $\frac{10}{100} + \frac{3}{100}$.

When we line up the decimal points we are actually changing the fraction $\frac{1}{10}$ to its equivalent form of $\frac{10}{100}$. It is easy because decimals are based on powers of 10 .

It is easy to change the appearance of fractions because the value of a fraction remains the same if it is multiplied by a form of the number one.

Example: To add $\frac{1}{2} + \frac{1}{3}$, we **must** change these to equivalent fractions which have the same denominator.



> Contrary to popular opinion, one does not get a special award for finding the "lowest" or "least" common denominator (LCD). Any common denominator will do. The advantage of using the smallest common measure only helps to keep our fractions in a form using smaller numbers; it is not "the only common measure".

One common denominator for this example is 6 , but you could use 12 , or others.

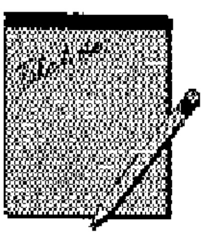
Rewriting the problem in a vertical form often helps to organize your work.

$$\begin{array}{r} \frac{1}{2} = \frac{?}{6} \\ + \frac{1}{3} = \frac{?}{6} \\ \hline \end{array} \qquad \begin{array}{r} \frac{1}{2} = \frac{3}{6} \\ + \frac{1}{3} = \frac{2}{6} \\ \hline \frac{5}{6} \end{array}$$

So, $\frac{1}{2} + \frac{1}{3}$ is equivalent to $\frac{3}{6} + \frac{2}{6}$ which equals $\frac{5}{6}$.

Example: $\frac{3}{8} + \frac{5}{12}$

The least common denominator is 24 , however, a common denominator can always be found by multiplying 8 and 12 . Hence, 96 is another common denominator.



> When taking a test with multiple choice answers, consider looking at the denominators in the answers for a possible common denominator.

$$\begin{array}{r} \frac{3}{8} = \frac{9}{24} \\ + \frac{5}{12} = \frac{10}{24} \\ \hline \frac{19}{24} \end{array}$$

Hence, the answer is $\frac{19}{24}$.

Example: $\frac{2}{7} + \frac{11}{14}$

In this example, note that one denominator is a **factor of the other**. When this is the case, use the **larger denominator** as the common denominator. This will require fewer steps since only one of the fractions will be changed.

$$\begin{array}{r} \frac{2}{7} = \frac{4}{14} \\ + \frac{9}{14} = \frac{9}{14} \\ \hline \frac{13}{14} \end{array}$$

Hence, the answer is $\frac{13}{14}$.

Now for subtraction. Again, and yes we know you must be really tired of hearing this, *you may only subtract the same units.*

Consider the following:

If we want to subtract 239 - 67 we would set it up as

$$\begin{array}{r} 239 \\ - \ \ 67 \\ \hline \end{array}$$

First we subtract the "ones". 9 "ones" minus 7 "ones" equals 2 "ones".

$$\begin{array}{r} 239 \\ - \ \ 67 \\ \hline 2 \end{array}$$

Next we attempt to subtract the "tens". 3 "tens" minus 6 "tens" . . .Uh oh! Problem! So we "borrow" (borrowing is a misnomer because we never give it back but actually just change the unit to an equivalent value!). We actually take 1 of the hundreds and change it into the equivalent number of tens.

1 "hundred" = 10 "tens", then add the 10 "tens" to the 3 "tens" we already have. Because it is in *base ten,* adding these is equivalent to putting a 1 in front of the 3 . So, we have 13 "tens". Now we can finish the problem.

$$\begin{array}{r} {}^{1}\not{2}{}^{1}39 \\ - \ \ 67 \\ \hline 172 \end{array}$$

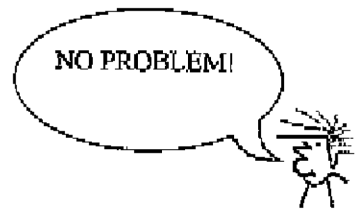
Subtraction of decimals is done in the same way. It is easy to do because when we "borrow" 1 from the larger unit it is always equivalent to **10** of the smaller unit.

Example:

$$\begin{array}{r} {}^{2}\not{3}.{}^{1}277 \\ - \ \ .865 \\ \hline 2.412 \end{array}$$

Fractions follow the same method. As with any addition problem the first step would be to change the problem into an equivalent problem with the same denominator.

Example: $\begin{array}{r} 3\frac{1}{2} \\ -\ 1\frac{1}{3} \\ \hline \end{array}$ would become $\begin{array}{r} 3\frac{3}{6} \\ -\ 1\frac{2}{6} \\ \hline 2\frac{1}{6} \end{array}$



Example: $\begin{array}{r} 3\frac{1}{3} \\ -\ 1\frac{1}{2} \\ \hline \end{array}$ would become $\begin{array}{r} 3\frac{2}{6} \\ -\ 1\frac{3}{6} \\ \hline \end{array}$

But, not if you understand "borrowing". We cannot subtract $\frac{3}{6}$ from $\frac{2}{6}$. So, we "borrow" from the "ones" unit (the next larger unit).

$3\frac{2}{6}$ becomes $2\frac{2}{6} + 1$. Now because this is not based on "10's", there is no easy way to add 1 to $\frac{2}{6}$ but we do know how to do this problem.

$1 + \frac{2}{6}$ $\Rightarrow$ because this is addition we may only add if they have the same denominator

So, $1 = \frac{6}{6}$.

Now the problem is $\frac{6}{6} + \frac{2}{6} = \frac{8}{6}$.

So, $3\frac{2}{6}$ becomes $2\frac{8}{6}$.

$\begin{array}{r} 2\frac{8}{6} \\ -\ 1\frac{3}{6} \\ \hline 1\frac{5}{6} \end{array}$ $\Rightarrow$ Now this is an equivalent form that we can do.

This answer seems correct. If we look at the original problem, $3\frac{1}{3} - 1\frac{1}{2}$ should be a little less than 2. To check, if there is time, we could add $1\frac{5}{6} + 1\frac{1}{2}$ and see if we get $3\frac{1}{3}$. Check this out.

To summarize this example:

$$\begin{array}{rcccc} & 3\frac{1}{3} & = 3\frac{2}{6} & = & 2\frac{8}{6} \\ - & 1\frac{1}{2} & = 1\frac{3}{6} & = & 1\frac{3}{6} \\ \hline & & & & 1\frac{5}{6} \end{array}$$

Now that all four of the basic operations for rational numbers have been demonstrated, the concept of **order of operations** can be introduced.

If a computation of rational numbers includes several basic operations, such as addition, subtraction, multiplication, and division, and perhaps includes some other concepts such as exponents and radicals (square roots), the final result depends on the **order** in which these operations are completed. It is important to follow these steps:

> Step 1 - Do the arithmetic that is inside sets of parentheses. Perform the operations inside the parentheses using the same rules for order of operations. (Treat the calculation inside each set of parentheses as if it were a separate problem.)
>
> Step 2 - Evaluate numbers that have exponents and also those that are expressed in radical form.
>
> Step 3 - Starting at the left, do the operations of multiplication and/or division in the order in which they occur going from left to right in the problem.
>
> Step 4 - Complete addition and/or subtraction from left to right in the order in which they occur.

Example: $8 \div 2^2 + (7^2 - 6 \times 3 \div 2) + 5 - (9 \times \sqrt{25})$

Step 1: $8 \div 2^2 + (\underline{7^2} - 6 \times 3 \div 2) + 5 - (9 \times \underline{\sqrt{25}})$ [The underlined part is to be done next.]

$8 \div 2^2 + (49 - \underline{6 \times 3} \div 2) + 5 - (\underline{9 \times 5})$

$8 \div 2^2 + (49 - \underline{18 \div 2}) + 5 - (45)$

$8 \div 2^2 + (\underline{49 - 9}) + 5 - (45)$

$8 \div 2^2 + (\quad 40 \quad) + 5 - (\ 45 \ )$

$8 \div \underline{2^2} + 40 + 5 - 45$

Step 2: $\underline{8 \div 4} + 40 + 5 - 45$

Step 3: $\underline{2 + 40} + 5 - 45$

Step 4: $\underline{42 + 5} - 45$

Step 5: $\underline{47 - 45}$.

Step 6: 2 [answer]

Simplify using the order of operations.

$3^2 + 4 \times (6 - 5) \div 2$

$16 \div (10 + 3 \times 2) + 0 \times 5^2$

Addition And Subtraction of Rational Numbers

and Order of Operations

Add or subtract each of the following:

1. $2.6 + 5.03$

2. $6 + 4\frac{1}{2}$

3. $4.56 - 1.3$

4. $14\frac{7}{8} - 5\frac{5}{8}$

5. $3.56 - 2.876$

6. $16\frac{2}{3} - 10$

7. $11\frac{1}{2} + 5\frac{2}{3}$

8. $3\frac{13}{16} - 2\frac{7}{8}$

9. $5\frac{1}{5} - 4\frac{1}{3}$

10. $3 + 12.235$

11. $12 - 8\frac{8}{11}$

12. $6\frac{4}{7} - 3\frac{6}{7}$

Simplify each of the following using the order of operations.

13. $3 + 9 \div 3$

14. $30 \div 6 - 12 \div 3$

15. $20 \div (3 + 4 - 2)$

16. $(16 - 2 \times 3\frac{1}{2}) \times 1.1$

17. $2\frac{1}{9} + 7\frac{1}{2} \times 2$

18. $6^2 \div 3$

19. $48 \div (2^3 + 4)$

20. $12 - (3 + 2^2) + 4.3$

21. $(3 + 9) \div 3$

22. $(2^4 + 4) \div 5$

23. $(2 + 3 \times 2.1) - 2.7$

24. $2.45 + 3.8 - \frac{7}{9} \times 0$

4.2 Percent of Increase or Decrease Problems

Problems that require using the concept of **increase** or **decrease** occur daily in the lives of all consumers. Knowing how to evaluate the different applications helps in making important decisions. Getting the most for your money is not being "cheap", it is being "SMART"!

These problems, often referred to as increase/decrease problems, may also be thought of as "**two-step**" percent problems since they most often require two basic steps in their solutions. The two steps would be the one to find the missing *percent, part(percentage), or whole(base),* and either an addition or subtraction step. Some sample applications would include:

1. Finding the final cost after tax (*percentage*) has been added to the original cost (*base*).
2. Finding the final cost after a discount (*percentage*) has been deducted from the original cost (*base*).
3. Determining the amount of tax (*percentage*) that has already been added to the original cost (*base*).
4. Determining the original cost (*base*) if the rate of discount (*percent*) and the discount (*percentage*) have been given.
5. Finding the percent of increase or decrease (*percent*) of a value that has changed.

In general, the ***original cost*** or the ***original value***, the one that occurs first, is the *whole(base)* in the problem. The ***percent of increase or decrease*** is the *percent*, and the actual ***increase or decrease*** is the *part(percentage)*.

A **modified "percent proportion"** follows:

$$\frac{\textit{percent of increase or decrease}}{100} = \frac{\textit{amount of increase or decrease}}{\textit{original amount}}$$

Percent becomes an extremely useful tool because it allows a common rule or group of computations to be applied to all values that could possibly be anticipated. Evaluating the final cost, including sales tax, of any purchase is a good example of this fact.

Example: Find the final cost of a refrigerator after a sales tax of 6% is applied (added). The original cost of the refrigerator is $900 .

[This kind of problem is one that is very common. Here are two methods that you can use to solve it.]

Method 1:

Step 1: First find the **amount of the tax** (*the amount of increase*).

You are given the *percent of increase* to be 6% , and you are also given the *original amount* of 900 . So, write your proportion.

$$\frac{6}{100} = \frac{x}{900}$$

Since $x = 54$, then the **amount of tax** is $54 .

Step 2: To determine the **final cost** you must **add** the tax to the original cost.

Hence, $900 + $54 = **$954** , the **final cost** of the refrigerator.

Method 2:

Step 1: Since the percent and the base are known, you can find the **amount of tax** by multiplying 900 by 0.06 .

Step 2: You would still need to **add** that product, 54 , to the 900 for the final cost which is $954 .

[An alternative to these two steps would be to first add 100% + 6% mentally, and then multiply 900 by 1.06 to get the final cost of $954 . When the original value is increased by a percent of it, the final value becomes the result of 100% + the percent of increase.]

Another common example, one that uses a percent to be **deducted** (subtracted), involves discounts on purchases.

Example: A coat that originally cost $200 is on sale. The discount is 25%. Find the final cost.

Step 1: First find the **amount of discount** (*amount of decrease*).

$$\frac{25}{100} = \frac{x}{200} \qquad \text{or} \qquad 0.25 \times 200 \qquad \text{or} \qquad \tfrac{1}{4} \text{ of } 200$$

would all yield the same result of 50.

Hence, the amount of discount (a percentage of the original cost) is $50.

Another way to say this is that the coat is on sale for "$50 off".

Step 2: To determine the final cost you would **subtract** the discount from the original cost.

Since $200 - 50 = 150$, then the final cost of the coat is \$150.

To describe the change that may occur over time of a value, it becomes useful to translate specific values into percents of the original value. Especially in situations where specific values are not necessary, and a general discussion of a trend or a comparison are needed, percents help to convey a clear picture.

Example: In September 1994, the number of students enrolled in math classes at G.C.C. was 1500. In September 1995, the number of students enrolled in math classes was 1605. Find the *percent of increase.*

[Keep in mind that in order to use the "percent proportion for increase/decrease problems", you'll need to know both the original amount **and** the amount of increase or decrease before attempting to find the missing percent.]

Step 1: First find the **amount of increase** by subtracting the original amount (1994's enrollment) from the new amount (1995's enrollment).

$1605 - 1500 = 105$, which represents the *amount of increase.*

Step 2: You now have enough information to complete your proportion with only one unknown value, the percent.

Remember: in a problem concerning percent of increase/decrease, the specific amount of change is **always** compared with the original amount or the amount that chronologically occurred first. So, put the value that occurred **first** (in 1994) in the place for the **original amount**.

$$\frac{x}{100} = \frac{105\ (\textit{amount of increase})}{1500\ \ (\textit{original amount})}$$

Since $x = 7$, then the percent of increase is 7%.

A Potpourri of Percent Problems

1. The retail sales tax rate in Florida is 4%. Find the total cost of a $3500 car.

2. Herman Hues, a paint salesman, has a 10% commission rate. What is his commission if he sells $4800 worth of paint?

3. A scratched refrigerator regularly priced at $460 was sold at a 10% discount. What was its new price?

4. The highest price paid at an auction for a vintage bottle of wine was $13,200. If the sales tax was 5%, how much tax was that?

5. The original price of a ping-pong paddle was $9.70, find the new price after a 20% price increase.

6. The sales tax on a car was $150, and the sales tax rate was 5%. What was the purchase price (before taxes) of the car?

7. A super soaker is on sale at 20% off its regular $15 price. What is the sale price?

8. Find the new price of a gold bracelet after a 210% price increase, if the original price was $32.00.

9. The retail sales tax rate in California is 6%. Find the total cost of a lava lamp selling for $15.50.

10. Jo Cool, an agent, books a band for $98,500. Her commission was $6860. What was the rate of commission?

11. A company had 66 fewer employees in July of 1995 than in July of 1994. If this represents a 5.5% decrease, how many employees did the company have in July 1994?

12. A stereo system is marked down from $450 to $382.50. What is the discount rate?

13. A school's enrollment was up from 950 students in one year to 1064 students in the next. What was the rate of increase?

4.3 Application Problems

As more and more consumers today are clipping coupons and shopping for the "best deal", keep in mind that effective and efficient "bargain shopping" requires a good working knowledge of percents. **Smart shoppers** should be aware of advertisements that may be misleading or misinterpreted, therefore they should "do their homework" when comparison shopping.

Example: Suppose a piece of luggage that regularly sold for \$79.99 is on sale for \$19.99 at a discount store with an accompanying ad that reads, "\$60 off". If the same item is for sale at a department store with the same regular price and the ad reads as follows, "All luggage---60% off!!", which store is offering the better deal? (Round the prices to the nearest dollar to make it easier.)

Since the amount of savings is already known for the discount store, calculate the amount of savings at the department store and then compare these amounts.

Discount store: Savings is \$60 .

Department store: Find 60% of \$80 to obtain the amount of savings.

60% of $80 = 48$, so the savings would be \$48 .

Hence, the discount store is offering the best deal.

Example: Suppose you are shopping at the mall and an item that interests you is marked "\$10 off". You assume that this is a 'real buy' since the item regularly sold for \$50 . But your thrifty shopping companion mentions that the same item is on sale at a store on the lower level for 25% off. If the regular price is the same at both stores, which store is offering the better deal?

One way to solve this problem would be to find the percent of discount at the upper level store and compare it to the 25% which is already known for the other store. Since \$10 is the *amount of discount (decrease),* a proportion can now be set up as follows: $\dfrac{x}{100} = \dfrac{10(\textit{amount of decrease})}{50(\textit{original amount})}$. Since $x = 20$, the percent of discount at this store is only 20% . Hence, the lower level store has the best deal. Can you think of another way to solve this problem?

Bargain Shopper

Consider the following items for sale at:

Cheapo's and Bargain Basement (B.B.)

In each case you will find the *sale price, best store* and the *difference in savings* between the two stores.

1. Bread Machine – regularly $249.99

On sale at **Cheapo's** for $50 off regular price — Cheapo's = ________

On sale at **B.B.** for 25% off regular price — B.B. = ________

Best Choice = ________

Savings Diff. = ________

2. Waffle iron – regular price $39.99

On sale at **Cheapo's** for 15% off regular price — Cheapo's = ________

On sale at **B.B.** for $5.00 off regular price — B.B. = ________

Best Choice = ________

Savings Diff. = ________

3. Supreme Cotton towels – regularly $8.99

On sale at **Cheapo's** for $33\frac{1}{3}$% off — Cheapo's = ________

On sale at **B.B.** for $6.49 — B.B. = ________

Best Choice = ________

Savings Diff. = ________

4. 16 piece beverage set – regular price $25.00

On sale at **Cheapo's** for $19.99 Cheapo's = ________

On sale at **B.B.** for 35% off regular price B.B. = ____________

Best Choice = ____________

Savings Diff. = ____________

If you wanted to purchase all four items at the same store, (because of time constraints) which store should you shop and what would your total cost be?

Sum of all four items: Cheapo's = ________

B.B. = ____________

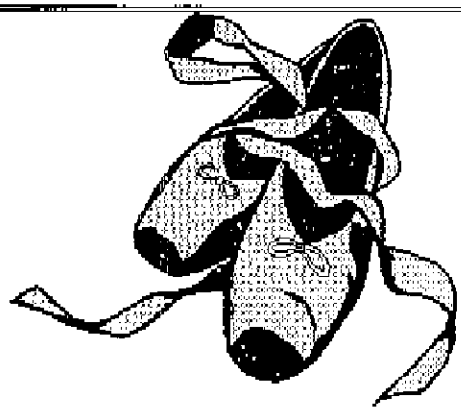
Best Choice = ____________

Savings Diff. = ____________

If the cheaper store is far away, over a toll bridge and with metered parking, for a total round trip cost of $15.00 and the other store is right up the street, would this change your choice?

Serious Student Shopper

The following are ads for Bargain Barn:

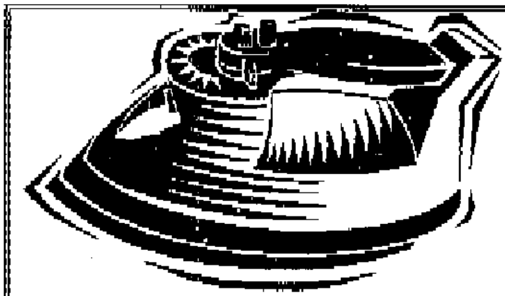


Save $7.50
Tiptoe Ballet shoes
Reg. $30

a) What is the sale price?

b) What is the percent saved?

c) Dance City has the same shoes for 30% off the regular price. Which store has the better deal?



$30
Solid iron Iron
All the whistles & bells
Reg. $49.99

a) What is the amount of savings?

b) What is the percent saved?

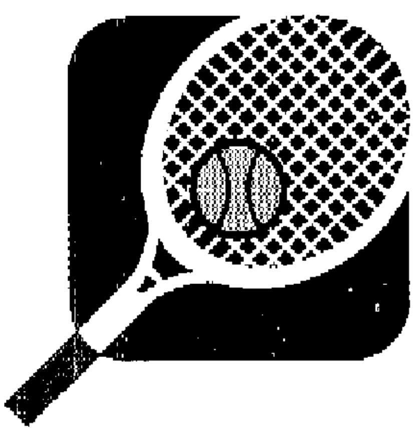
c) Cheap Mart had the same item for 25% off the regular price. Which store has the better deal?

a) What is the sale price?

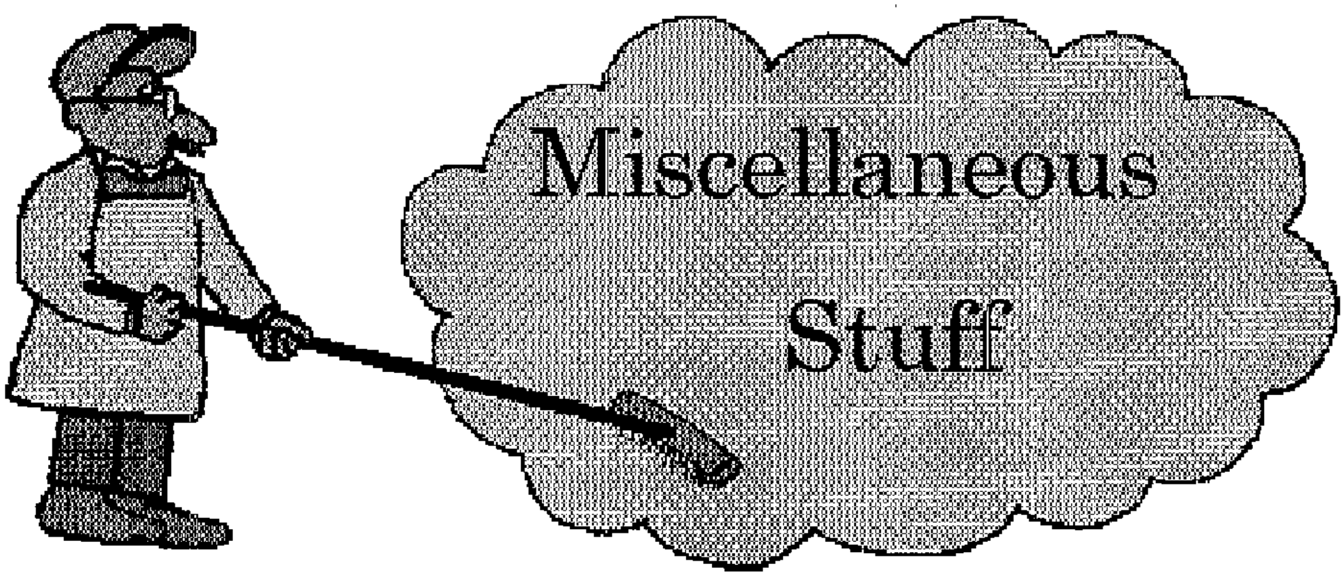
b) What is the amount of savings?

c) Phone World has the same phone for $\frac{1}{3}$ off. At which store should shrewd consumer shop?

If Hobby Hut has these same skates for 60% off the regular price, which store has the best deal?



Bargain Barn has all their closeout tennis equipment on sale for 50% off the **original price** and on one special day gives another 10% off the already **reduced price**. What percent of the **original price** is the consumer actually saving?


Miscellaneous
Stuff

Test Taking, Study, and Problem Solving Strategies Summary

Problem solving strategies

1. Read the entire problem before attempting any activity.
    *focus on what you are asked to find
    *do not make up your own problem
    *use self-talk to guide you
    *estimate the answer
    *check your answer by comparing to your estimate
    *re-read the question to make sure the answer you have is to the question being asked
    *try to rely on understanding instead of rules
    *mark any problems that you need help with

Study strategies

1. Studying math is practicing problems.
    *make sure you have answers for the problems you do
    *practice checking your answers by estimation
    *mix-up the kinds of problems you practice (flash cards might help)
    *use self- talk while practicing
    *time yourself when practicing
    *if possible practice the same time of day as the test will be given
    *look back at quizzes , homework , or classwork for practice problems
    *break up your study time, study more frequently but for shorter periods
    *work with a buddy, give each other problems to do
2. Find out about the test.
    *what is the test format (multiple choice)
    *how will it be graded
    *how many problems
    *how much time

Test taking strategies

1. Put yourself in the best mental frame.
    *use relaxation techniques
    *picture yourself doing well
    *think of the test as showing what you *can* do
2. Follow the directions.
3. Review with yourself time constraints and decide how much time to allow for each problem.
    *If you can't start a problem or can't finish within your time, mark the problem and come back later.
    *for multiple choice tests, if you can rule out any answers do so then come back later
    *after you have done the problems you *can* do, go back and work on the ones you had difficulty with
4. Solve the problems. (see above.)

Review

1. $(0.03)^2 =$ ________

2. The price of a \$24 math book was reduced by $33\frac{1}{3}\%$. How much does it cost now? ________

3. Write $1 + \frac{3}{1000} + \frac{6}{10{,}000}$ as a decimal. ________

4. $32.3 - 5.07 =$ ________

5. Write $\frac{1}{2}\%$ as a decimal. ________

6. $130\overline{)3.939} =$ ________

7. What is 20 percent of 20 ? ________

8. If 4 boxes of pencils cost \$3.08 , how much will 7 boxes cost? ________

9. $\frac{8}{9} + \frac{2}{5} - \frac{1}{3} =$ ________

10. Which of the following is the closest approximation to 17.001×2.2222 ?

    a. 35,000 b. 350 c. 35 d. 0.0035 ________

11. $\frac{8}{0.016} =$ ________

12. Stephanie found some money on the sidewalk. If she spent $\frac{2}{5}$ of the money for books and $\frac{1}{4}$ for a new radio, what fraction of her money did she have left? ________

13. $0.74 + \frac{1}{4} =$ ________

14. 7 is what percent of 28 ? ________

15. $\sqrt{0.0016} =$ ________

16. $8\frac{1}{5} \times 4\frac{3}{4} =$ ________

17. 12 boys in the senior class are on the track team. If this is 5% of the boys in the class, how many boys are in the class? ________

18. $\dfrac{12}{\frac{3}{7}} =$ ________

19. Which of the following is the closest to $\sqrt{20000}$? ________

a. 200 b. 100 c. 140 d. 400

20. If 28% of the 250 employees at the local building supply store worked overtime hours for the last week, how many employees worked **regular** hours for that week? ________

Answers: 1. 0.0009 2. $16 3. 1.0036 4. 27.23 5. 0.005 6. 0.0303 7. 4 8. $5.39 9. $\frac{43}{45}$ 10. c 11. 500 12. $\frac{7}{20}$ 13. 0.99 14. 25% 15. 0.04 16. $38\frac{19}{20}$ 17. 240 18. 28 19. c 20. 180

f T * E * S * T

Count the number of times the letter f appears in the paragraph below. You have two minutes to finish. You may not mark the page.

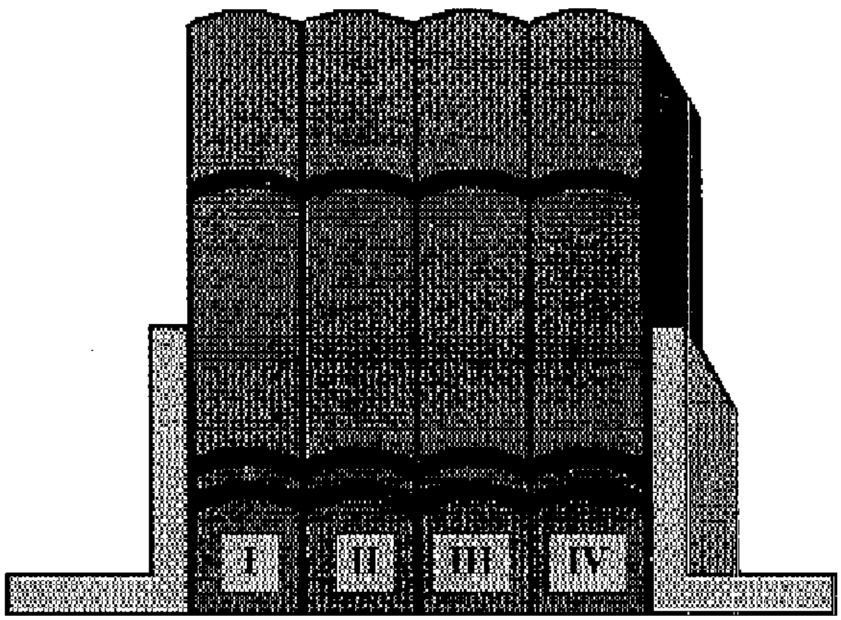
> The necessity of training farm hands for first-class farms in the fatherly handling of farm live stock is foremost in the minds of effective farm owners. Since the forefathers of the farm owners trained the farm hands for first class farms in the fatherly handling of live stock, the farm owners feel they should carry on with the former family tradition of training farm hands of first-class farms in the effective fatherly handling of farm live stock, however futile, because of their belief it forms a basis of effective management efforts.

Answer: page 173

Book Worm Problem

While this problem looks deceptively simple, it is actually quite difficult. As a matter fact, only about one person in a hundred is able to solve it the first time around. The problem is included because it is extremely instructive.

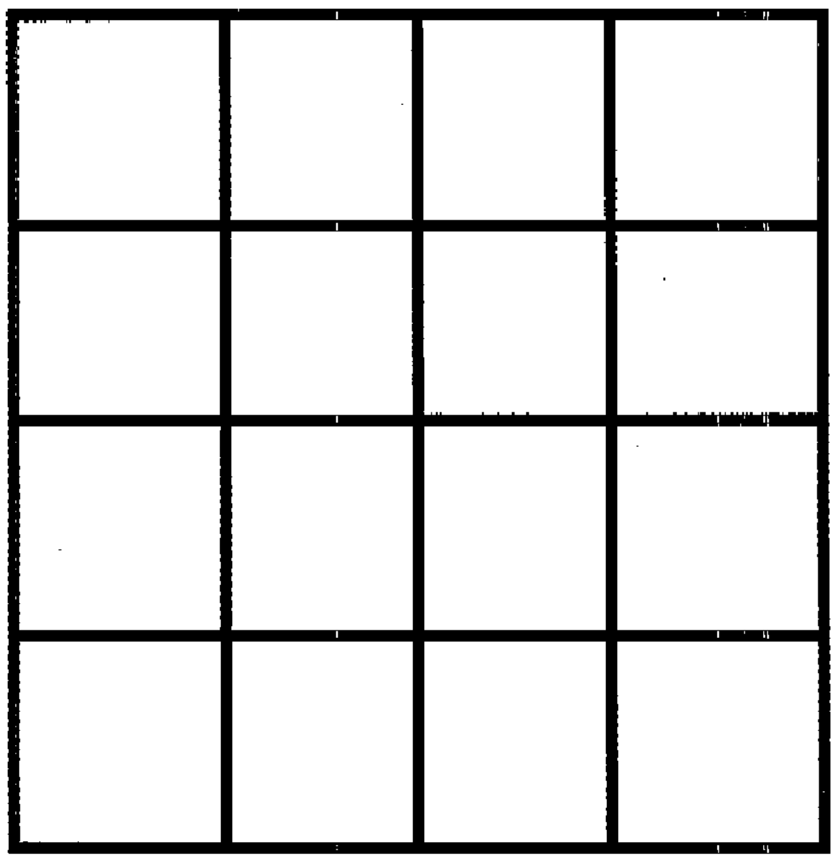
There are four volumes of Shakespeare's collected works on the shelf. The pages of each volume are exactly 2″ thick. The covers are each $\frac{1}{6}''$ thick. The bookworm started eating at page 1 of volume I and ate through to the last page of volume IV. What is the distance the bookworm traveled?



Answer: page 173

How many squares are there in the following diagram?

Hidden Squares Figure

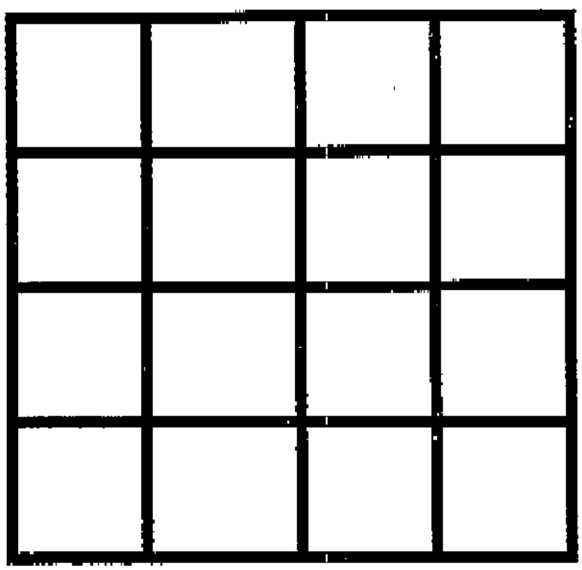


Answer: page 173

Answers and Suggestions

1. There are 46 'f's' in the "f test". Turn the paper upside down and then count the letter 'f' shapes without reading the words.

2. The bookworm traveled 5 inches. Notice the position of the first page of Volume I and the last page of Volume IV in the diagram.

3. 

Key: The correct answer is 30, developed as follows: 1 whole square, 16 individual squares, 9 squares of 4 units each, and 4 squares of 9 units each.

Discussion Questions:

1. What factors prevent us from easily obtaining the correct answer?
   (We stop at the first answer. We work too fast.)
2. How is this task like other problems we often face?
   (Many parts comprise the whole.)
3. What can we learn from this illustration that can be applied to other problems?

Magic? Recipe for Succeeding in Math

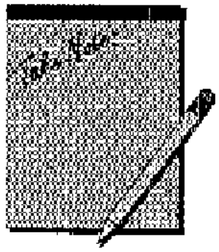
Make sure you are where you belong.

Attend all classes and be prompt.

Get assignments done for the next class so you'll be ready to progress.

Improve your understanding by asking questions. Get involved by working with other classmates.

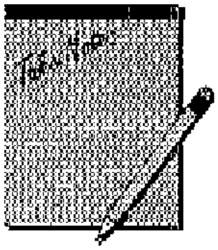
Connect-recognize how math concepts are connected to each other and how the concepts connect to daily life.



Dear Student,

You may use the form below to take notes. It is suggested that you write the problems in the left column and any helpful thoughts in the right column.

Photocopy this sheet, if needed.



Dear Student,

You may use the form below to take notes. It is suggested that you write the problems in the left column and any helpful thoughts in the right column.

Photocopy this sheet, if needed.

Card Games

Fish: (1) Deal 7 cards to each player (three or more players); place the remaining cards in a draw pile.

(2) Each player takes a turn asking player to the left for a "match" to complete a "book". A "book" consists of 3 cards that are equivalent forms (fraction, decimal, percent) of a rational number.

(3) a. If the player on the left does have a "match", she gives it to the first player. The first player may then lay down a complete "book" of 3 equivalent cards, if possible. It is still the first player's turn, so she can ask the next player for a "match".

b. If the player on the left does *not* have a "match", he says "Go fish." At this time, the original player must draw a card from the draw pile. If this card is a match, the player may lay down a complete "book" of 3 equivalent cards, if possible. It is still the first player's turn, so she can continue to ask for cards. If the drawn card is not a match, play passes to the next player.

(4) The game ends when the draw pile is depleted, and the player with the greatest number of "books" wins.

War: (1) Play with two players.

(2) Divide the deck in half.

(3) Keep each pile face down in front of each player.

(4) Both players simultaneously turn one card face up and compare them. The player with the larger valued card takes both of the cards and places them in a separate pile to be used when his original pile is depleted. If the two cards are equivalent, both players simultaneously turn 3 cards (W-A-R) face down. They then proceed to each turn a 4th card face up and compare their values. The player with the larger valued card takes all 8 cards.

(5) The game ends when one player runs out of cards or if allotted time has ended. The player who acquired the greatest number of cards wins.

Up and Down the River: (Play with 3 or more players.)

Hand 1: Deal out 10 cards to each player.

Each player, beginning with the dealer, bids the number of *tricks* that he thinks he will win ranging from 0 - 10. (A player wins a trick if he has the highest card of the suit being played.)

The last person to bid may not bid the number of tricks that would allow <u>everyone</u> to make what they bid.

Example: Player one says "4", player two says "2", ... player three may bid anything **except** 4 .

Dealer leads, and the suit (fraction, decimal, or percent) must be followed, if possible.

Example: If a fraction is lead, players must follow with fractions.

If a player cannot follow suit, he may play anything but he can't take the trick.

Score. (See below.)

Hand 2: Deal out 9 cards to each player. (Follow procedure for hand 1 but base it on 9.)

Remaining hands: Deal out 8,7,6,5,4,3,2,1, then 1,2,3,4,5,6,7,8,9,10 for a complete game, or go as far as time allows.

Scoring: At the end of each hand every player gets 1 point for each trick he takes in. He also receives 10 bonus points if he makes exactly the number of tricks he said he would.

Additional games: Players can make up their own versions of matching games such as "Concentration".